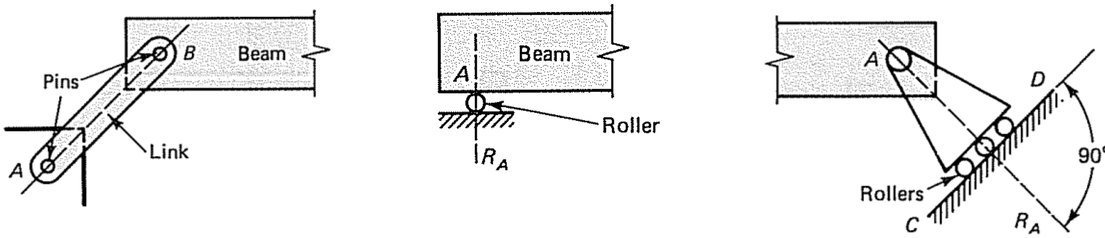
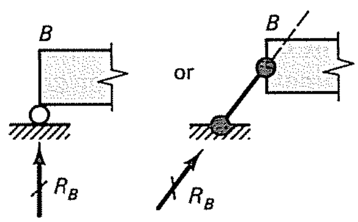
Axial force, Shear, and Bending Moment

Diagrammatic Conventions for Supports: three types of supports are recognized for planar structures:

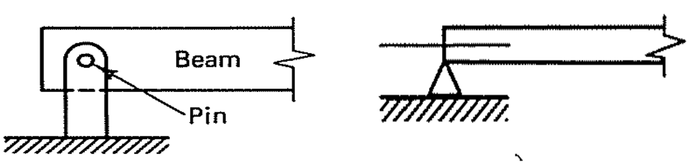
1- Roller or Link support: It is capable of resisting a force in only one specific line of action. The link can resist a force only in the direction of line AB, the roller can resist only a vertical force, whereas the rollers can resist only a force that acts perpendicular to the plane CD



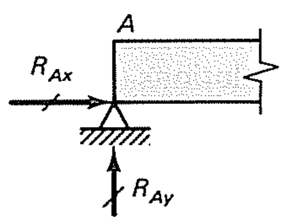
(a) (b) (c)



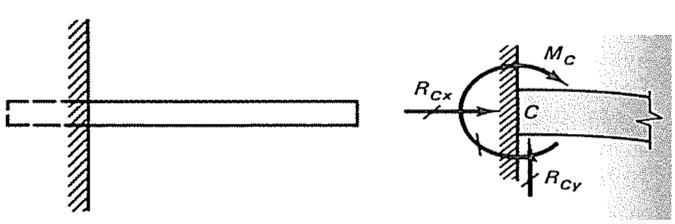
2- Pin or Hinge support: is capable of resisting force acting in any direction of the plane. Hence, in general, the reaction at such a support may have two components, one in the horizontal and one in the vertical direction.



(a) (b)

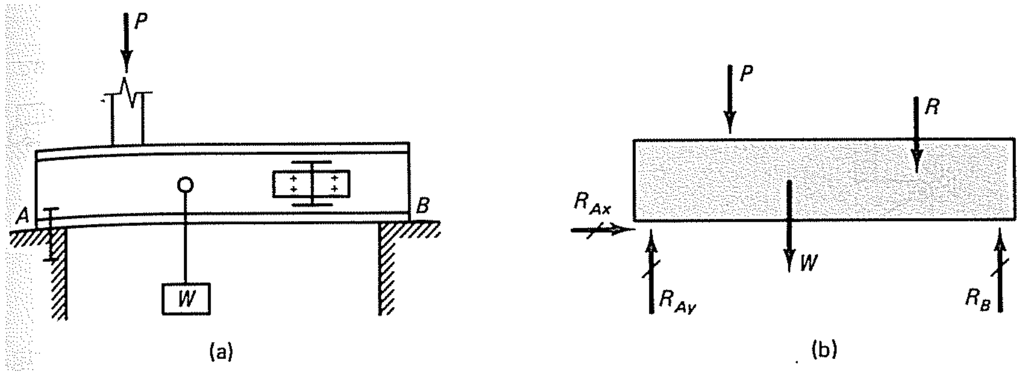


3- Fixed support: is able to resist a force in any direction is also capable of resisting a moment or a couple.

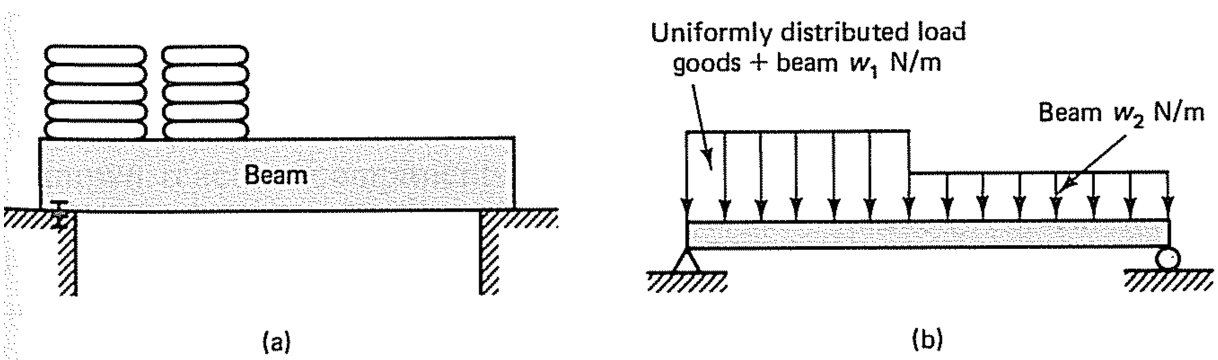


Diagrammatic Conventions for Loading:

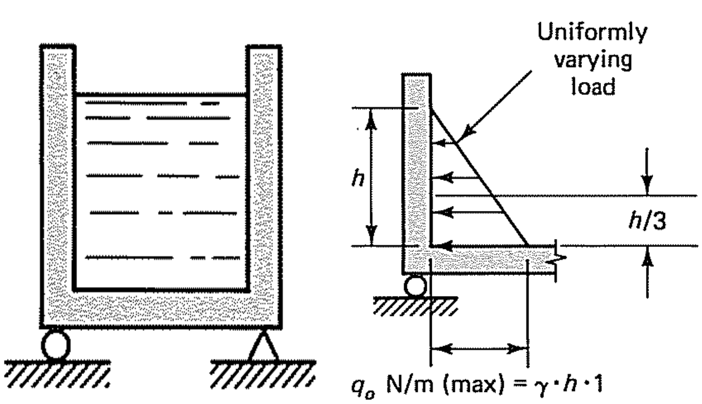
1- Concentrated loads: a force is applied to a beam through a post, a hanger or a bolted.



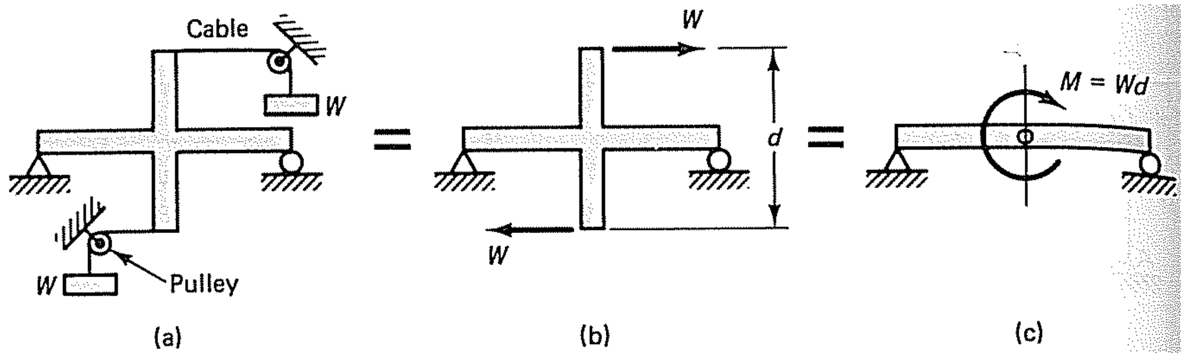
2- Uniformly distributed loads: This load is usually expressed as force per unit length of the beam it may be given as newton per meter (N/m)



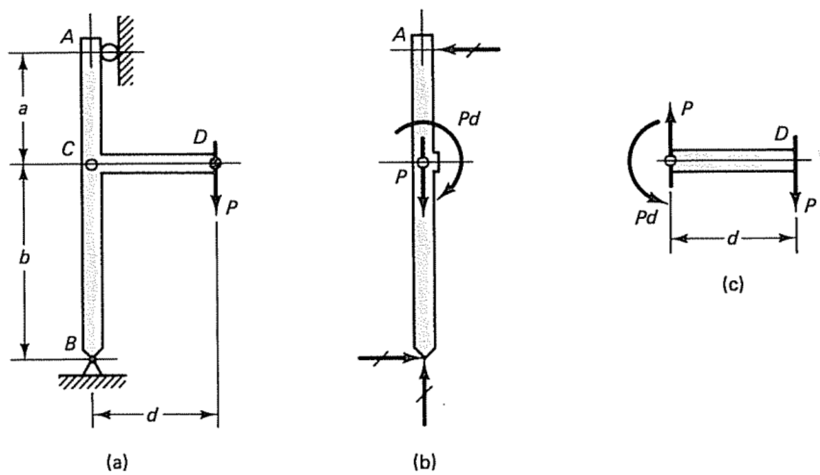
3- Uniformly varying loads: act on the vertical and inclined walls of a vessel containing liquid γ (N/m^3) is the unit weight of the liquid, the maximum intensity of the load of q_0 N/m, the total force exerted by such a loading on a beam is ($q_0h/2$) N, and its resultant acts at a distance *h/3* above the vessel's bottom.



It is conceivable to load a beam with a concentrated moment applied to the beam essentially at a point. One of the possible arrangements for applying a concentrated moment is shown in Figure below:

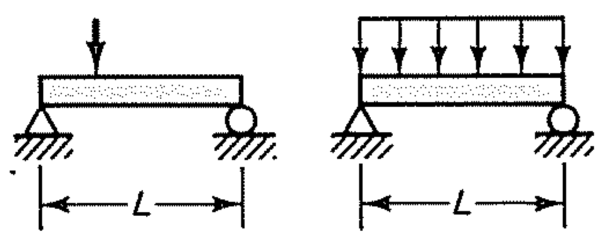


In order to maintain the applied force *P* in equilibrium at joint C, a shear *P* and a moment *Pd* must be developed at the support. These forces apply a concentrated moment and an axial force, as shown in Figure

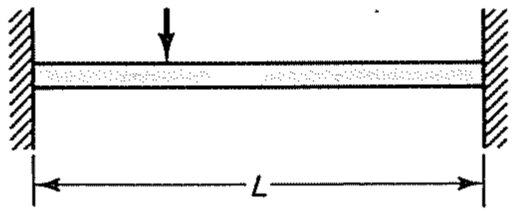


Classification of Beams: Beams are classified into several groups, depending primarily on the kind of supports used:

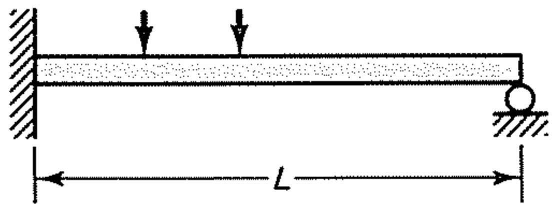
1- Simply supported or simple beam: if the supports are at the ends and are either pins or rollers.



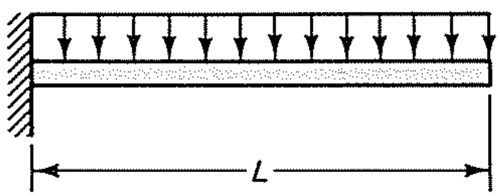
2- Fixed beam or fixed-ended beam: if the ends have fixed supports.



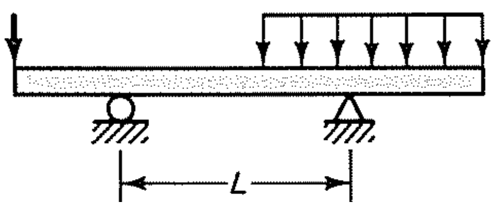
3- Restrained beam: a beam fixed at one end and simply supported at the other.



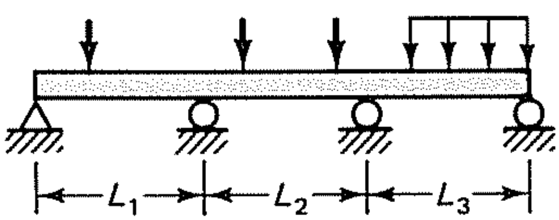
4- Cantilever beam: a beam fixed at one end and completely free at the other



5- Overhanging beam: if the beam projects beyond a support, the beam is said to have an overhang



6- Continuous beam: is an intermediate supports are provided for a physically continuous member acting as a beam.



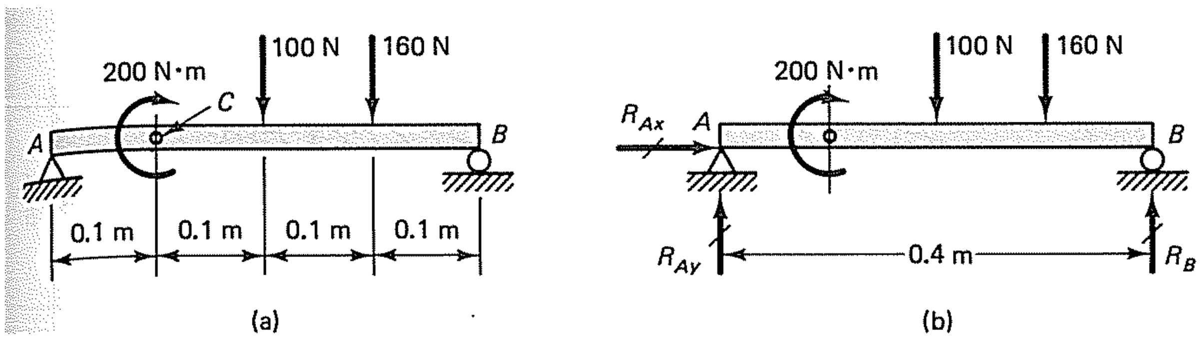
For all beams, the distance *L* between supports is called a span

Calculation of Beam Reactions

Three equations of static equilibrium are available for the analysis these are:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \text{and} \quad \sum M_z = 0,$$

Ex1: Find the reactions at the supports for a simple beam loaded as shown in Figure. Neglect the weight of the beam.



$$\sum F_x = 0 \qquad \rightarrow \qquad R_{Ax} = 0$$

$$\sum M_A = 0 + \qquad \rightarrow \qquad 200 + 100 \times 0.2 + 160 \times 0.3 - R_B \times 0.4 = 0$$

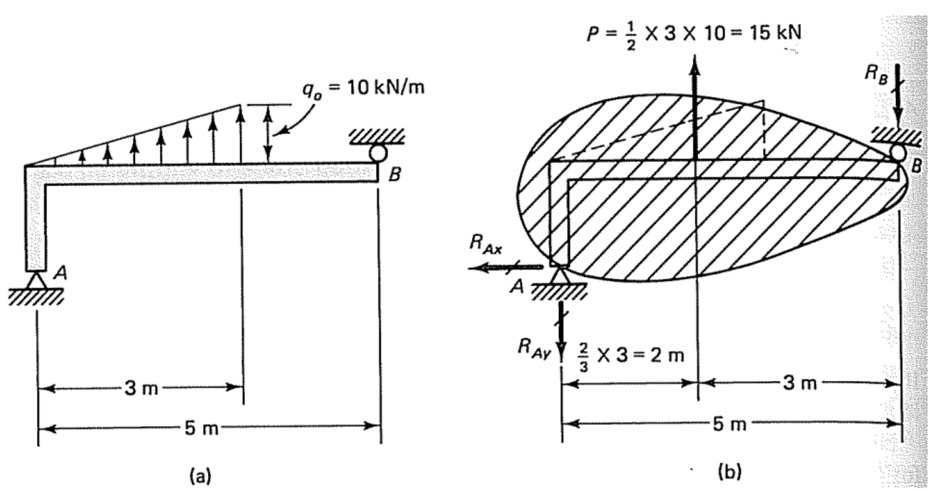
$$R_B = +670N \uparrow$$

$$\sum M_B = 0 + \quad \rightarrow R_{Ay} \times 0.4 + 200 - 100 \times 0.2 - 160 \times 0.1 = 0$$

$$R_{Ay} = -410N \downarrow$$

Check: $\sum F_y = 0 \uparrow + \qquad \rightarrow \qquad -410 - 100 - 160 + 670 = 0$

Ex2: Find the reactions for the partially loaded beam with a uniformly varying as shown in Figure. Neglect the weight of the beam.



$\sum F_x = 0 \qquad \rightarrow \qquad R_{Ax} = 0$

$\sum M_A = 0 \;\circlearrowleft + \qquad \rightarrow \qquad +15 \times 2 - R_B \times 5 = 0$

$$R_B = 6kN \downarrow$$

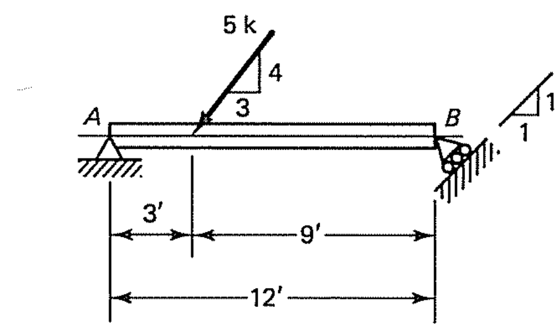
$\sum M_B = 0 \;+ \qquad \rightarrow \qquad -R_{Ay} \times 5 + 15 \times 3 = 0$

$$R_{Ay} = 9kN \downarrow$$

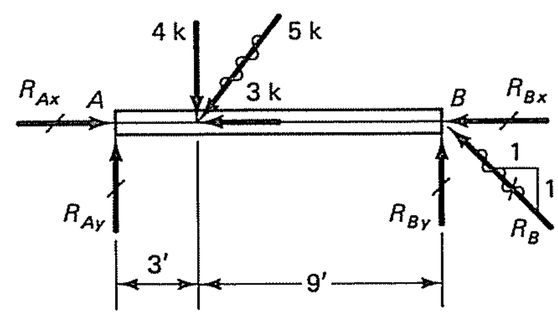
Check: $\sum F_y = 0 \uparrow += 0 \uparrow +$

$$-9 + 15 - 6 = 0$$

Ex3: Determine the reactions at A and B for the beam shown in Figure due to the applied force.



(a)



(b)

$\sum M_A = 0 \quad \rightarrow \quad +4 \times 3 - R_{By} \times 12 = 0$

$R_{By} = 1k \uparrow = |R_{Bx}|$

$\sum M_B = 0 \quad \rightarrow \quad +R_{Ay} \times 12 - 4 \times 9 = 0$

$R_{Ay} = 3k \uparrow$

$\sum F_x = 0 \rightarrow + \quad \rightarrow \quad R_{Ax} - 3 - 1 = 0$

$R_{Ax} = 4k \rightarrow$

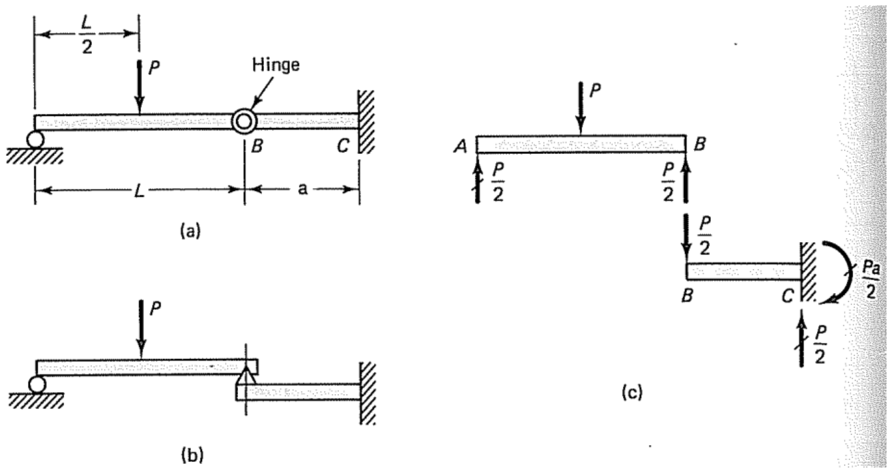
$R_A = \sqrt{4 + 3^2} = 5k$

$R_B = \sqrt{1^2 + 1^2} = \sqrt{2}k$

Check: $\sum F_y = 0 \uparrow +$

$+3 - 4 + 1 = 0$

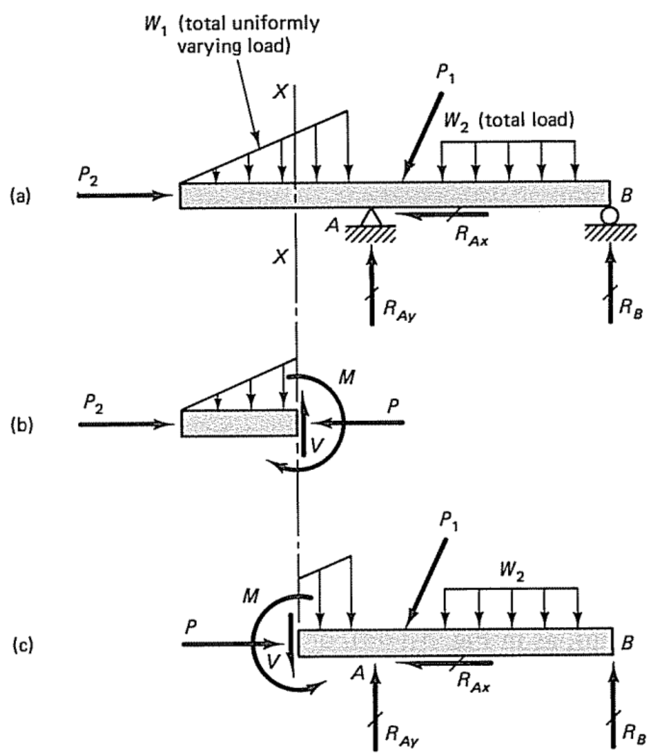
Occasionally, hinges or pinned joints are introduced into beams and frames. A hinge is capable of transmitting only horizontal and vertical forces. No moment can be transmitted at a hinged joint. Therefore, the point where a hinge occurs is a particularly convenient location for separation of the structure into parts for purposes of computing the reactions as shown in figure below:



Direct Approach for Axial Force Shear and Bending Moment

Application of the Method of Sections

The analysis of any beam or frame for determining the internal forces begins with the preparation of a free-body diagram showing both the applied and the reactive forces. The reactions can always be computed the equations of equilibrium provided the system is statically determinate. The method of sections can then be applied at any section of a structure by employing the previously used concept that if a whole body is in equilibrium, any part of it is likewise in equilibrium. Now consider an imaginary cut *X*-*X* normal to the axis of the beam, which separates the beam into two segments figures b&c.

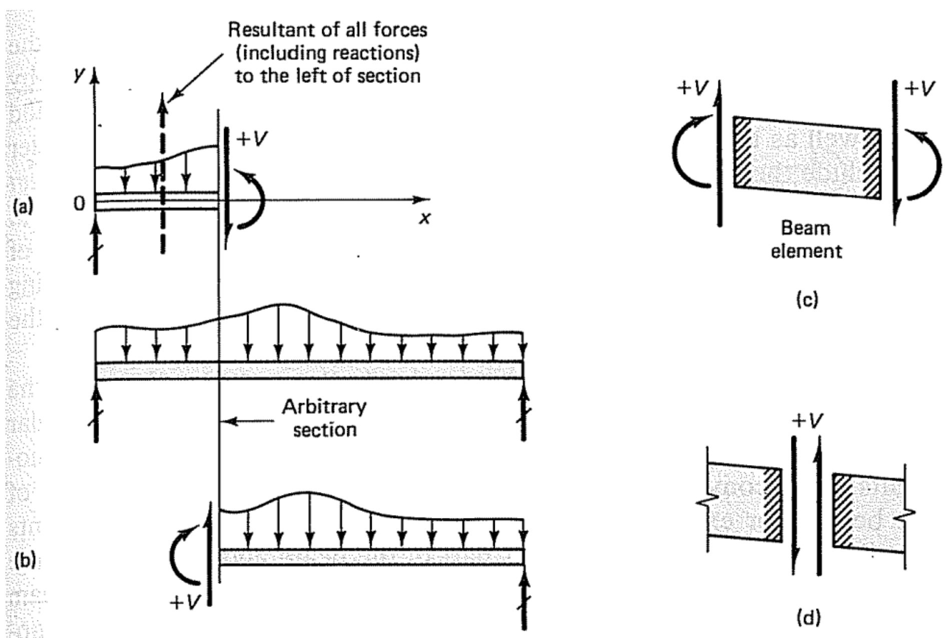


Axial Force in Beams

A horizontal force such as P, shown in Figure, may be necessary at a section of a beam to satisfy the conditions of equilibrium. The magnitude and sense of this force follows from a particular solution of the equation $\sum F_x = 0$. If the horizontal force P acts toward the section, it is called a thrust; if away, it is axial tension. It was shown that it is imperative to apply this force through the centroid of the cross sectional area of a member to avoid bending.

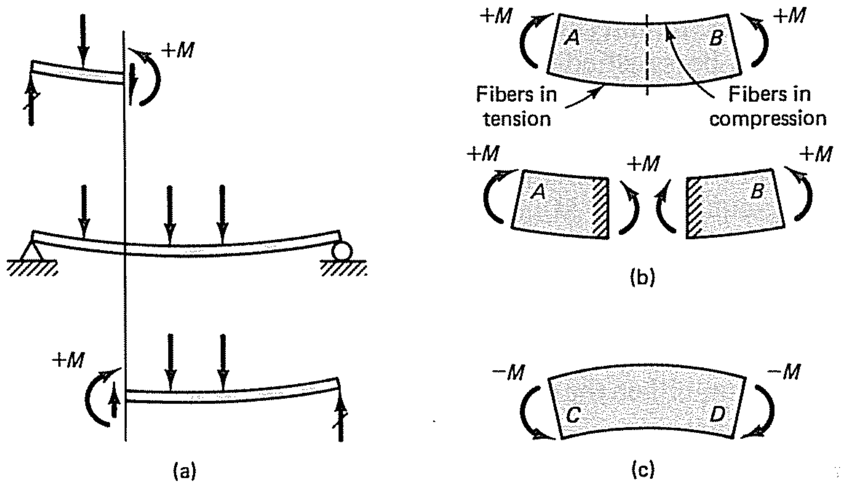
Shear in Beams

In general, to maintain a segment of a beam, such as that shown in figure in equilibrium, there must be an internal vertical force V at C cut to satisfy the equation $\sum F_y = 0$. This internal force V, acting at right angles to the axis of the beam, is called the shear, or shear force. The shear is numerically equal to the algebraic sum of all the vertical components of the external forces acting on the isolated segment, but it opposite in direction.

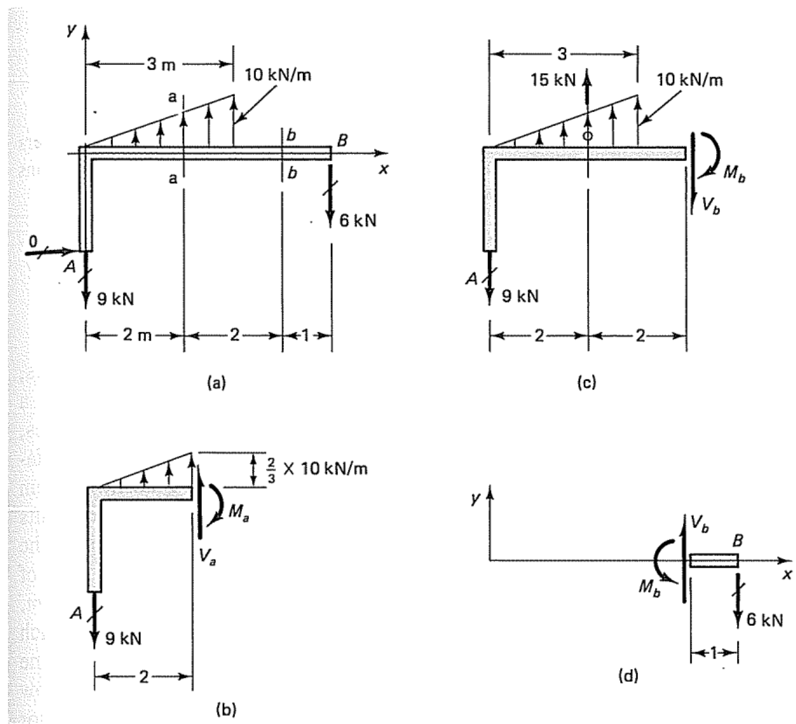


Bending Moment in Beams

The internal resisting moment must act in a direction opposite to the external moment to satisfy the governing equation$\sum M_z = 0$. It follows from the same equation that the magnitude of the internal resisting moment equals the external moment. These moments tend to bend a beam in the plane of the loads and are usually referred as bending moments.



Ex4: Consider earlier Example 2 and determine the internal system of forces at sections $a-a$ and $b-b$;



Solution

$$V_a = -9 + \frac{1}{2} \times 2 \times \frac{2}{3} \times 10 = -2.33kN$$

$$M_a = -9 \times 2 + \frac{1}{2} \times 2 \times \frac{2}{3} \times 10 \times \frac{1}{3} \times 2$$

$$= -13.61kN.m$$

and

$$V_b = +6kN$$

$$M_b = -6 \times 1 = -6kN.m$$

Axial-Force, Shear, and Bending-Moment Diagrams

By the methods discussed before, the magnitude and sense of axial ford shears, and bending moments may be obtained at many sections of beam. Moreover, with the sign conventions adopted for these quantities, a plot of their values may be made on separate diagrams.

Ex5: Construct axial-force, shear, and bending moment diagrams for the beam shown in Figure (a) due to the inclined force $P = 5\ k$.

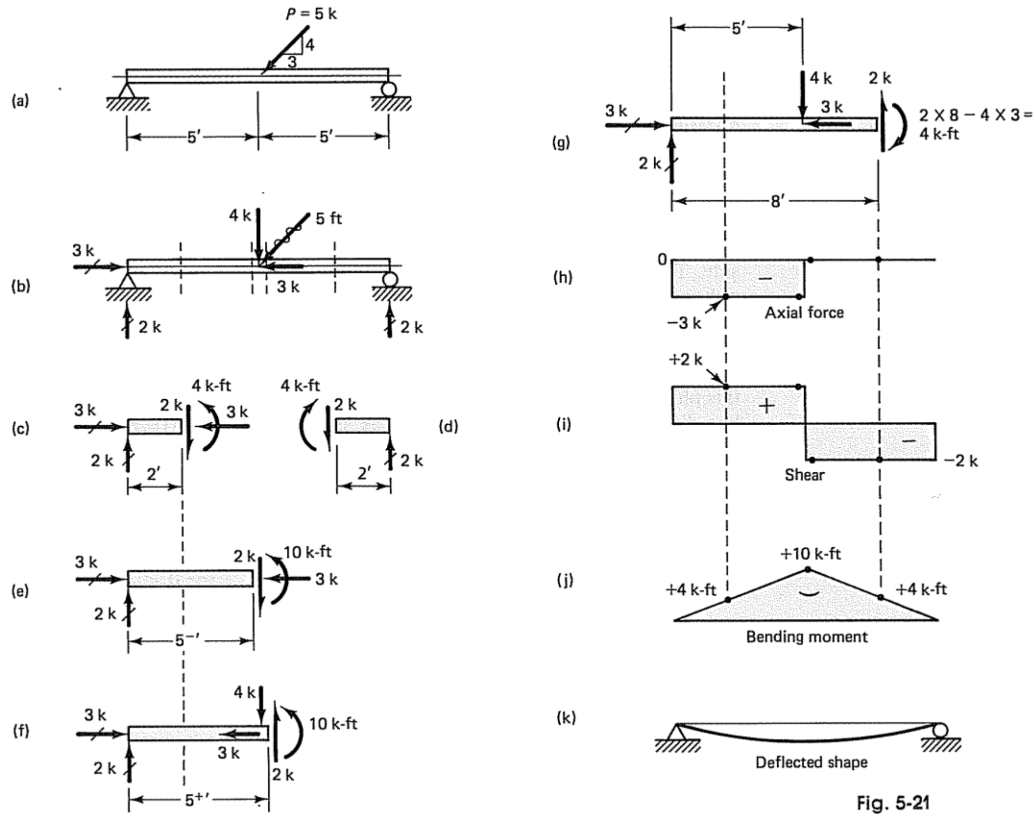


Fig. 5-21

Instead of the shear or moment diagrams, analytical expressions for these functions are necessary. For the origin of x at the left end of the beam, the following relations apply:

$V = +2k$ for $0 < x < 5$

$V = -2k$ for $5 < x < 10$

$M = +2x$ k-ft for $0 \le x \le 5$

$M = +2x - 4(x-5) = +20 - 2x$ k-ft for $5 \le x \le 10$

Ex6: For the beam in Example 4, shown in Figure, express the shear V and the bending moment M as a function of x along the horizontal member.

The required expressions for $0 < x < 3$ are

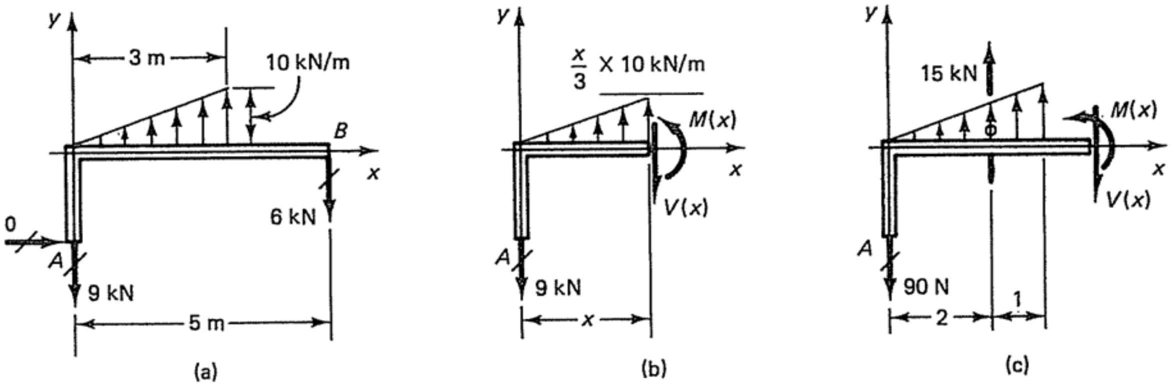
$$V(x) = -9 + \frac{1}{2}\chi(\frac{x}{3} \times 10) = -9 + \frac{5}{3}x^2 kN$$

$$M(x) = -9x + \frac{1}{2}x(\frac{x}{3} \times 10)(\frac{x}{3}) = -9x + \frac{5}{9}x^3 kN \cdot m$$

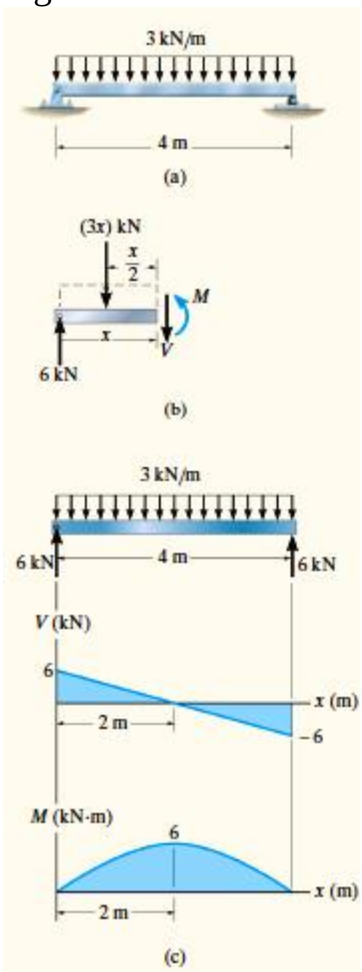
For $3 < x < 5$,

$$V(x) = -9 + 15 = +6kN$$

$$M(x) = -9x + 15(x-2) = 6x - 30kN \cdot m$$



Ex7: Draw the shear and moment diagrams for the beam shown in Figure.



$\sum F_y = 0 \quad \uparrow +$ *6 kN - (3x) kN - V = 0*

V= (6-3x)kN *(1)*

$\sum M_z = 0 \;\circlearrowleft +$ *-6kN(x) + (3x) kN (1/2x) + M = 0*

M= (6x - 1.5x^2) kN.m *(2)*

The point of zero shear can be found from Eq. 1:

V = (6 - 3x) kN = o

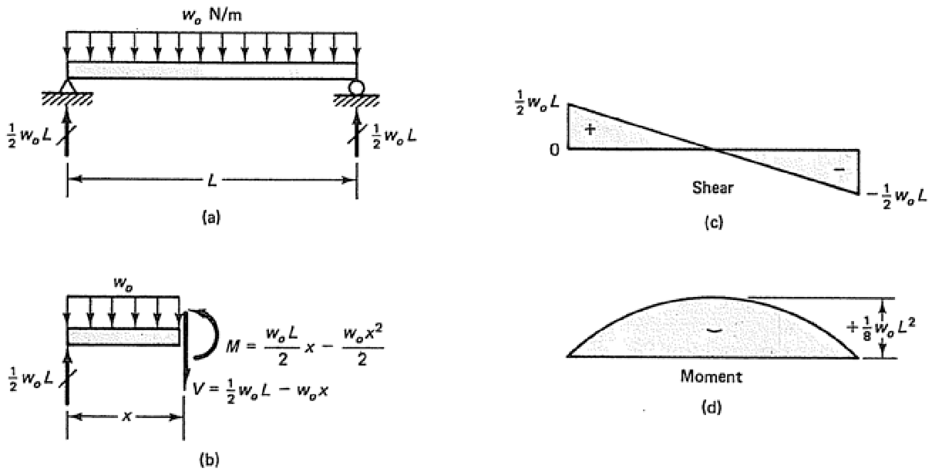
x = 2m

The maximum moment can be found from Eq. 2

M_{max} *= 6(2) - 1.5 (2)^2 kN.m*

= 6 kN. m

Ex8: Plot shear and a bending-moment diagrams for a simple beam with a uniformly distributed load.



Ex9: Draw the shear and moment diagrams for the beam shown in figure.

$0 \leq x_1 < 5m.$

$\sum F_y = 0 \uparrow +$ $\quad$ $5.75kN\text{-}V=0$

$V = 5.75\ kN$ $\quad$ (1)

$\sum M_z = 0\ \circlearrowleft +$ $\quad$ $-80kN.m—5.75kNx_1+M=0$

$M = (5.75x_1 + 80)\ kN.m$ $\quad$ (2)

$5m < x_2 \leq 10m$

$\sum F_y = 0 \uparrow +$ $\quad$ $5.75kN - 15kN - 5kN/m(x_2 - 5m) - V=0$

$V = (15.75 - 5x_2)kN$ $\quad$ (3)

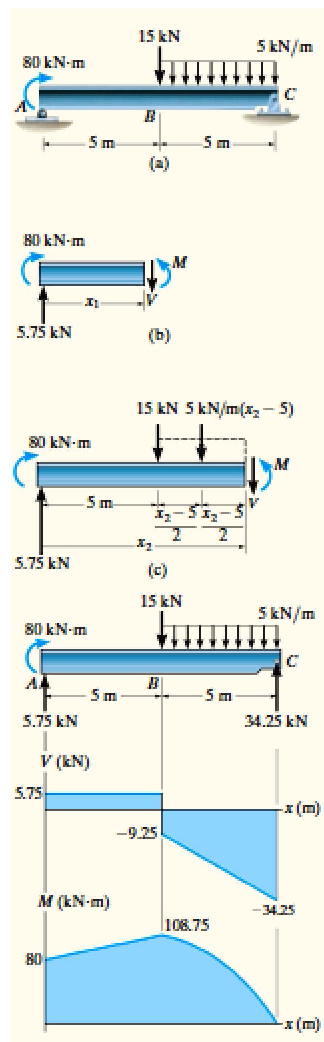
$\sum M_z = 0\ \circlearrowleft +$ $\quad$ $-80kN.m-5.75kNx_2+15kN(x_2-5m)$

$+5kN/m(x_2 - 5m)\ (\frac{x_2 - 5m}{2}) + M = 0$

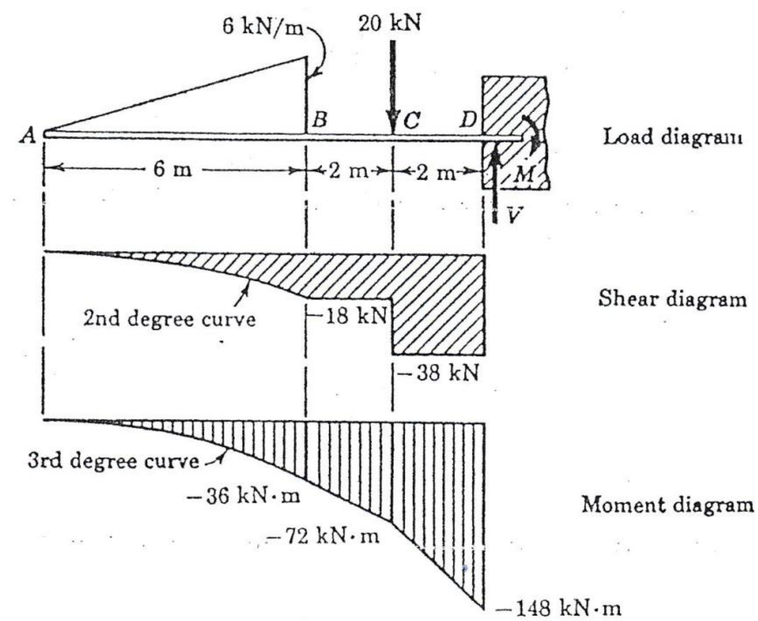
$M = (-2.5x_2^{\ 2} + 15.75x_2 + 92.5)kN.m$ $\quad$ (4)

Shear and Moment Diagrams Equations 1 through 4

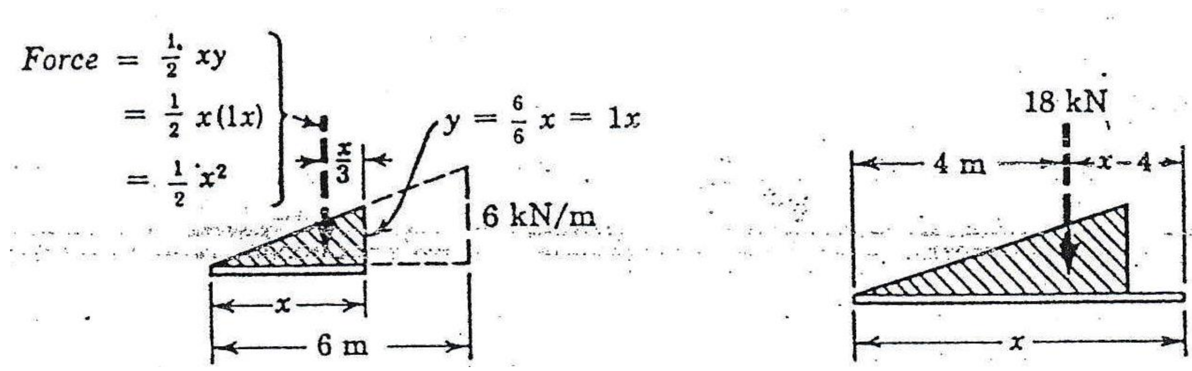
are plotted in Figure.



Ex10: write the shear arid moment equation for the cantilever beam carrying the uniformly varying load and concentrated load shown in Figure. Also sketch the shear and moment diagrams



For the region A B, in .which x varies from 0 to 6



$[V = (\Sigma\, F_y)_L]$ $\qquad V_{AB} = -\dfrac{x^2}{2} kN$

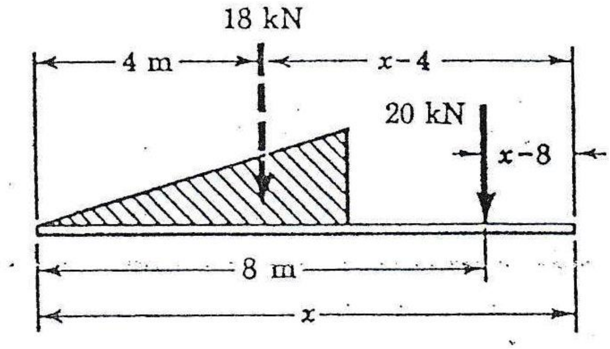
$[M = (\Sigma M)_L]$ $\qquad M_{AB} = -\frac{x^2}{2}(\frac{x}{3}) = -\frac{x^3}{6} kN \cdot m$

For the region BC, in which x varies between 6 and 8, we therefore obtain

$[V = (\Sigma\, F_y)_L]$ $\qquad V_{BC} = -18kN$

$[M = (\Sigma M)_L]$ $\qquad M_{BC} = -18(x-4\rangle = (-18x + 72)kN \cdot m$

For a section between C and D in which x varies from 8 to 10, we obtain

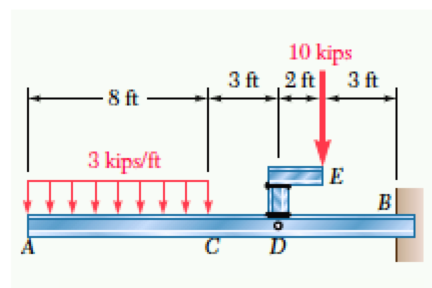


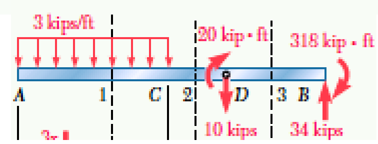
$[V = (\Sigma\, F_y)_L]$ $\quad V_{CD} = -18-20 = -38kN$

$[M = (\Sigma M)_L]$ $\quad M_{CD} = -18(x-4) - 20(x-8) = (-38x + 232)kN \cdot m$

$[M = (\Sigma M)_L]$ $\quad M_D = -18(6) - 20(2) = -148\ kN \cdot m$

Ex11: The structure shown consists of a rolled-steel beam AB and two short members welded together to the beam. Draw the shear and bending-moment diagrams for the beam.





From A to C

$V = \sum F_y$ $\quad V = -3x \quad kips$

$M = \sum M_z$ $\quad M = -1.5x^2 \quad kip.ft$

From C to D

$V = \sum F_y$ $\quad V = -24\ kips$

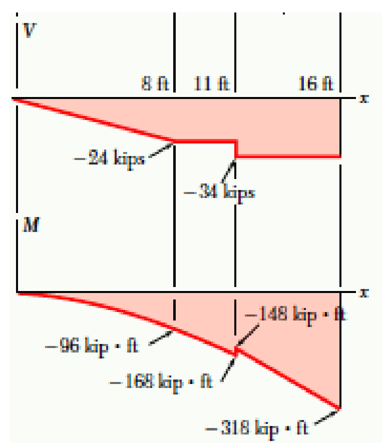
$M = \sum M_z$ $\quad M = -24(X-4) \qquad M = 96 - 24x\ kip.ft$

From D to B

$V = \sum F_y$ $\quad V = -24 - 10 \qquad V = -34\ kips$

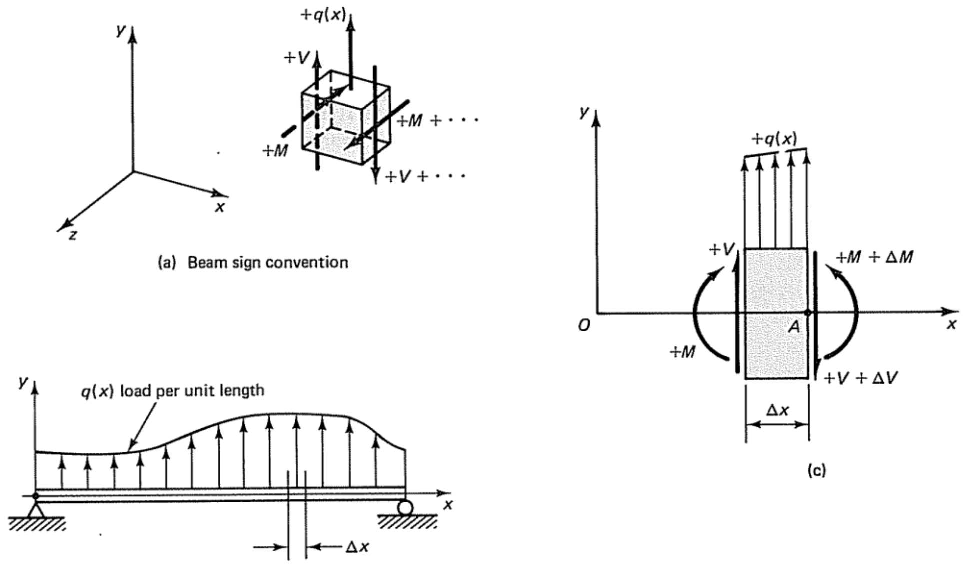
$M = \sum M_z$ $\quad M = -24(X-4) + 20 - 10(X-11) \qquad M = 226 - 34x\ kip.ft$

The shear and bending-moment diagrams for the entire beam now can be plotted.



Shear and Bending Moments by Integration

Consider a beam element Δx long, isolated by two adjoining sections taken perpendicular to its axis, Figure (b). Such an element is shown a free-body in Figure (c). All the forces shown acting on this element have positive sense. The positive sense of the distributed external force q is taken to coincide with the direction of the positive y axis.



(a) Beam sign convention
(b)
(c)

As the shear and the moment may each change from one section to the next, note that on the right side of the element, these quantities we, respectively, designated

$$V + \Delta V \qquad and \qquad M + \Delta M$$

From the condition for equilibrium of vertical forces, one obtains

$$\Sigma F_y = 0 \uparrow + \qquad V + q\Delta x - (V + \Delta V) = 0$$

$$\frac{\Delta V}{\Delta x} = q \qquad (1)$$

For equilibrium, the summation of moments around A also must be zero. So, upon noting that from point A the arm of the distributed force is $\Delta x/2$, one has

$$\sum M_A = 0 + \qquad (M + \Delta M) - V\Delta x - M - (q\Delta x)(\Delta x l 2) = 0$$

$$\frac{\Delta M}{\Delta x} = V + \frac{q\Delta x}{2} \qquad (2)$$

Equations 1 and 2 in the limit as $\Delta x \to 0$ yield the following two basic differential equations:

$$\frac{\Delta V}{\Delta x} = q \qquad (3) \qquad \text{and} \qquad \frac{\Delta M}{\Delta x} = V \qquad (4)$$

By substituting Eq. 4 into Eq. 3, another useful relation is obtained:

$$\frac{d}{dx}\left(\frac{dM}{dx}\right) = \frac{d^2M}{dx^2} = q \qquad (5)$$

By transposing and integrating Eq. 3 gives the shear *V*:

$$V = \int_0^x q\,dx + C_1$$

Slope of shear diagram $\frac{dV}{dx} = q$ $\rightarrow +Slope$, $\rightarrow -Slope$

Transposing and integrating Eq. 4 gives the bending moment

$$M = \int_0^X V\,dx + C_2$$

Slope of shear diagram $\frac{dM}{dx} = V$ $\rightarrow +Slope$, $\rightarrow -Slope$

<u>**OR**</u> integrating Eq. 3 we obtain:

$$dV = q dx$$

$$\int_{V_1}^{V_2} dV = \int_{x_1}^{x_2} q\,dx$$

$$V_2 - V_1 = \Delta V = (Area\ of\ load)$$

Similarly integrating Eq. 4 gives:

$$dM = V dx$$

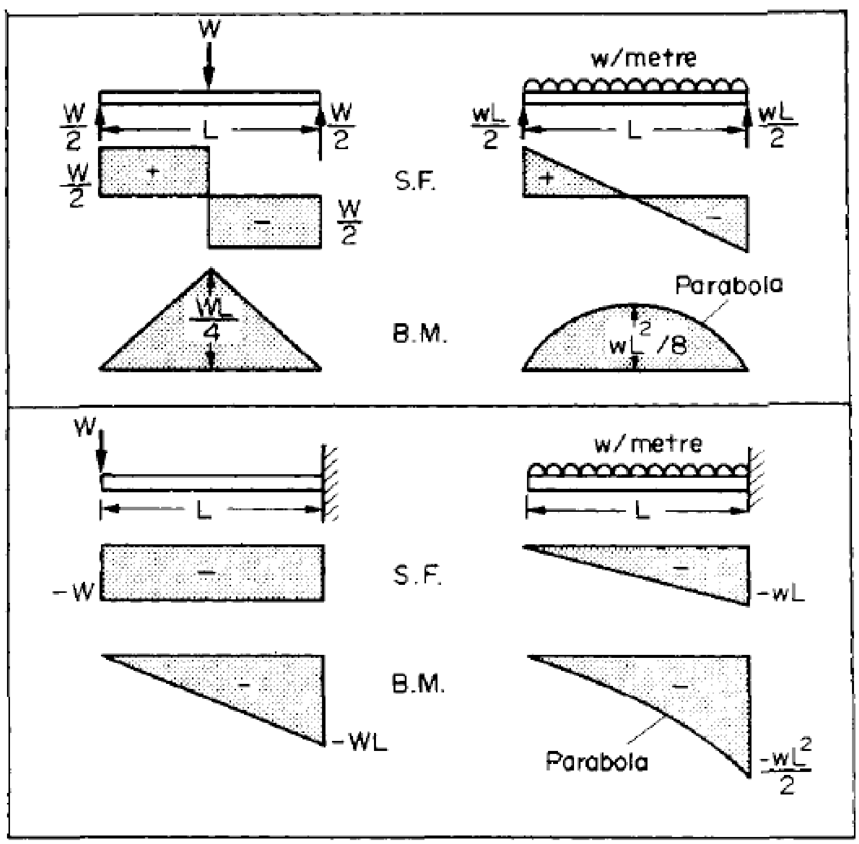
$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V\,dx$$

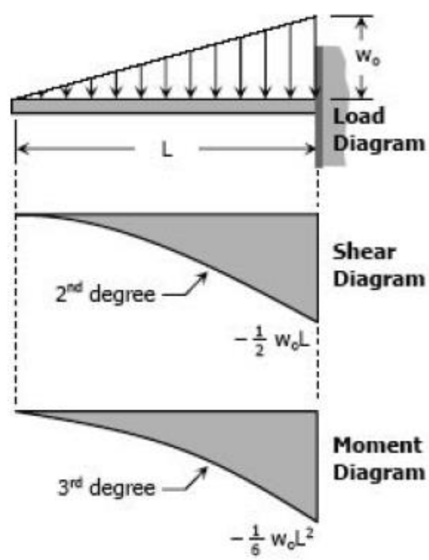
$$M_2 - M_1 = \Delta M = (Area\ of\ shear)$$

Relation between Loading, Shear Force and Bending Moment

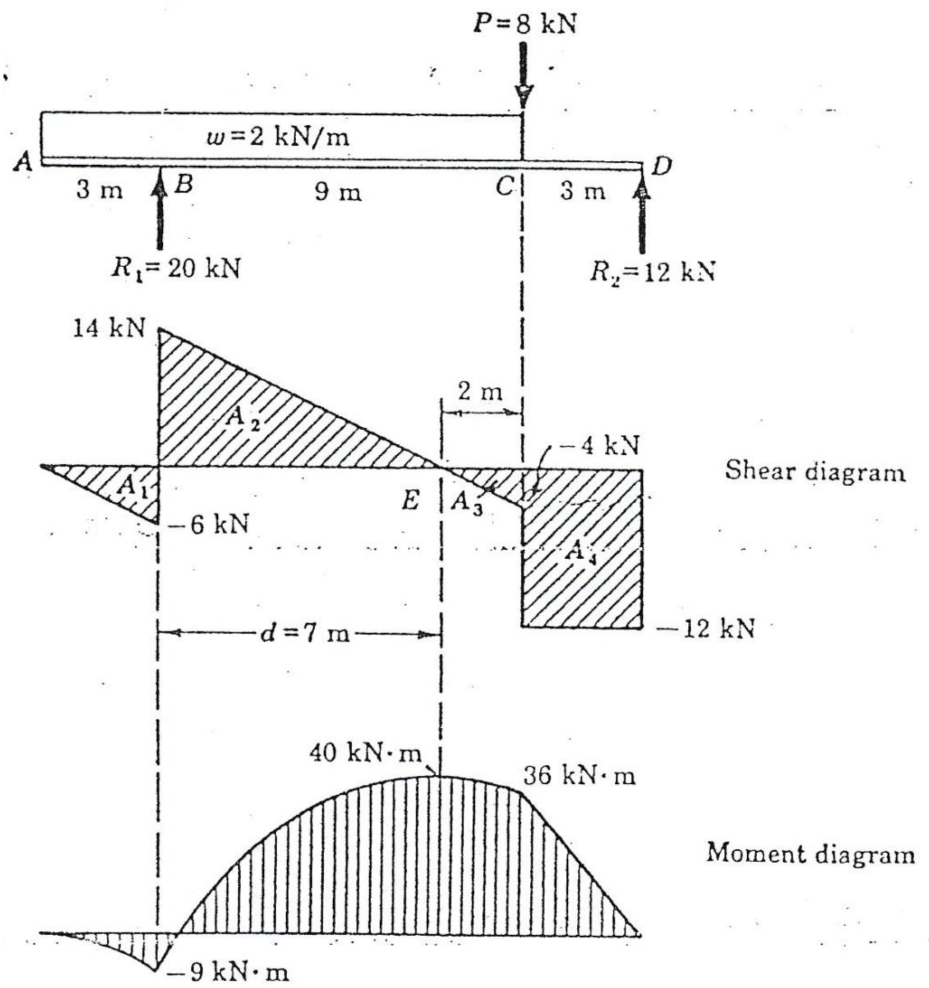
The following relations between loading, shear force and bending moment at a point or between any two sections of a beam are important from the subject point of view:

1- If there is a point load at a section on the beam, then the shear force suddenly changes (the shear force line is vertical). But the bending moment remains the same.
2- If there is no load between two points, then the shear force does not change (shear force line is horizontal). But the bending moment changes linearly (bending moment line is an inclined straight line).
3- If there is a uniformly distributed load between two points, then the shear force changes linearly (shear force line is an inclined straight line). But the bending moment changes according to the parabolic law. (Bending moment line will be a parabola).
4- If there is a uniformly varying load between two points then the shear force changes according to the parabolic law (shear force line will be a parabola). But the bending moment changes according to the cubic law.





Ex12: sketch shear and moment diagrams for the beam shown in Figure below, computing the values at all change of loading points and the maximum shear and maximum moment.



$\sum M_C = 0$ *and* $\sum M_B = 0$ → *R_1 = 20kN* *and* *R_2=12kN*

Shear diagram:

At the left of B, from the eq. $V_2 - V_1 = \Delta V = (Area\ of\ load)$ → *ΔV= -2 × 3= -6 kN*

Between B and C, the area of the load *-2 ×9 = 18 kN*

The concentrated load reaction at B causes the shear at B to increase abruptly by 20 kN to a net positive shear ordinate of 14kN at the right of B

The net shear at the left of C *V_C= V_B + Δ V = 14 - 18 = -4 kN.*

The concentrated load of 8kN changes the shear ordinate to -12 kN at the right of C.

The shear ordinate stays constant at this value between C and D

At D, the upward reaction of 12 kN reduces the shear ordinate to zero.

The interval BE at the rate of 2kN/m $\rightarrow$ *BE =d= 14/2=7 m.*

Moment diagram:

As a preliminary to computing the bending moment, we determine the areas of the shear diagram marked A_1, A_2, A_3, *and* A_4.

$$A_1 = \left(\frac{1}{2}\right)(3)(-6) = -9kN.m$$

$$A_2 = (\frac{1}{2})(7)(14) = +49kN.m$$

$$A_3 = \left(\frac{1}{2}\right)(2)(-4) = -\dot{4}kN.m$$

$$A_4 = 3(-12) = -36kN \cdot m$$

$From\ the\ eq.$ $\quad M_2 - M_1 = \Delta M = (Area\ of\ shear)$

The bending moment at B is given by A_1, M_B= - 9 kN.m

The bending moment at E is $M_E = M_B + \Delta M = A_1 + A_2$= -9 + 49= +40 kN.m

The bending moment at C can also be computed as the sum of the areas A_1, A_2, *and* A_3, *giving: Mc = 36 kN.m*

The moment at C changes to become zero at D:

M_D= Mc+ΔM = Mc+A_4 = 36 kN.m $-36kN.m = 0$