كلية المصطفى الجامعة

Fundamental of Electrical Engineering

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11-12-13-14-15 lec.

Thevenins theorem :

The current flowing through a load resistance R_L connected across any two terminals A and B of a network as shown in fig. 1 is given by :

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

Where V_{th} is the open circuit voltage across the two terminals A and B where R_L is removed .

R_{th} is the internal resistance of the network as viewed back into the network from terminals A and B with voltage source replaced by its internal resistance , while current source replaced by open circuit .

R_L load resistor .

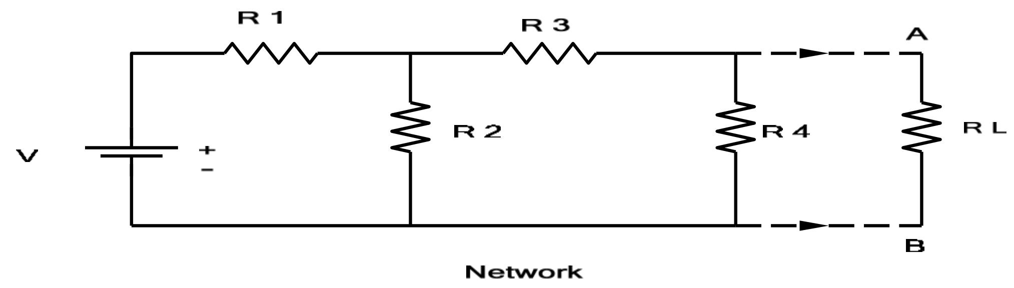
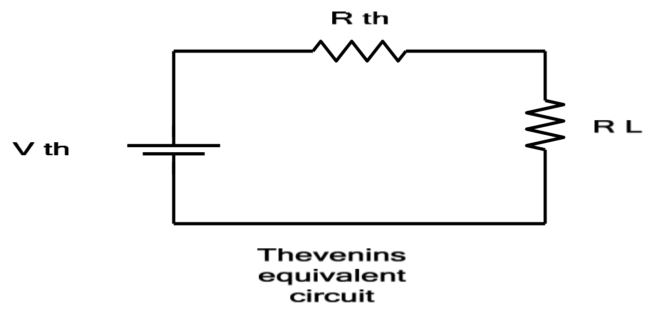


Fig. 1



- **Now , R_{th} and V_{th} must be found .**

R_{th} could be found as follows :

1.Replace voltage source by short circuit (if there is no internal resistance) , while the current source replaced by open circuit .

2.Remove R_L from the circuit , then calculate R_{th} viewed from terminals A and B .

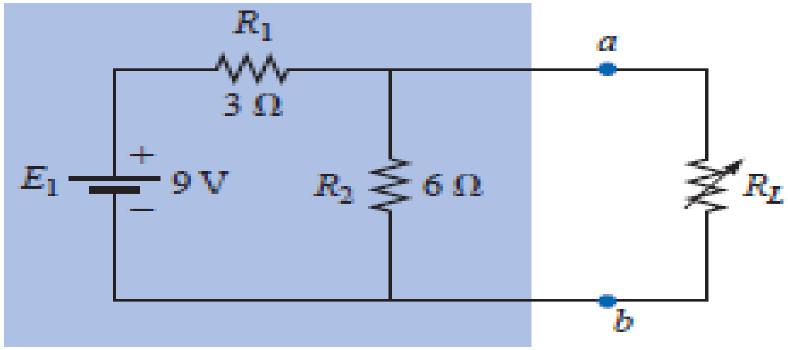
V_{th} could be found as follows

1. Remove R_L and make sure that the voltage or current source is connected .

2. Calculate V_{th} between points A and B .

<u>Example</u> :

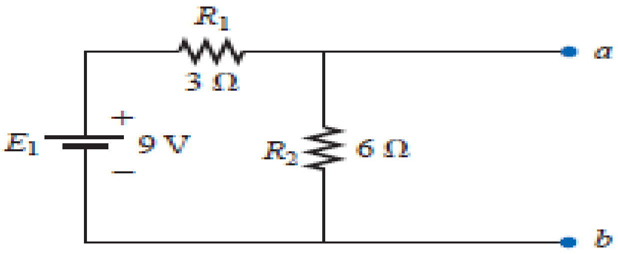
Find the Thévenin equivalent circuit for the network in the shaded area of the network of Figure Then find the current through *RL* for values of 2 , 10 , and 100:

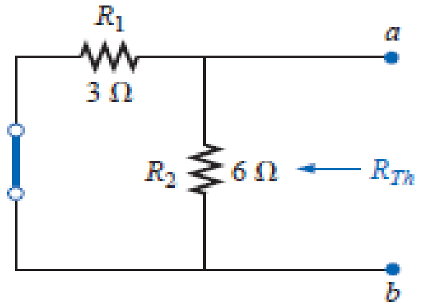


To find R_{th}

Steps 1 and 2 produce the network of Figure Note that the load resistor RL has been removed and the two "holding" terminals have been defined as a and b.

Step 3: Replacing the voltage source E1 with a short-circuit equivalent yields the network of Figure.



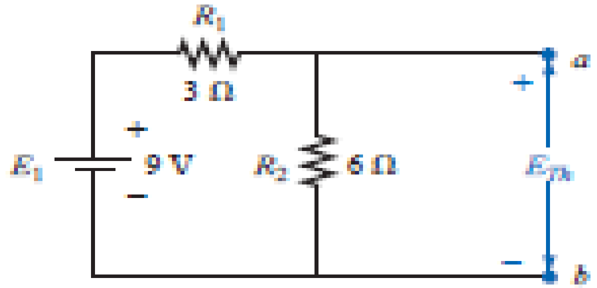


Then:

$$R_{Th} = R_1 \parallel R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = 2\ \Omega$$

To find V_{th}

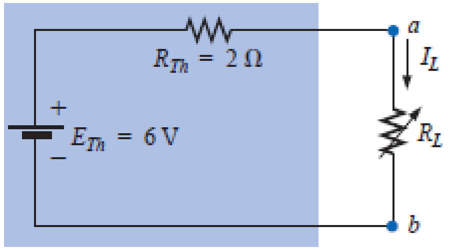
Step 4: Replace the voltage source (Figure. For this case, the open circuit voltage ETh is the same as the voltage drop across the 6- resistor.



Applying the voltage divider rule:

$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$



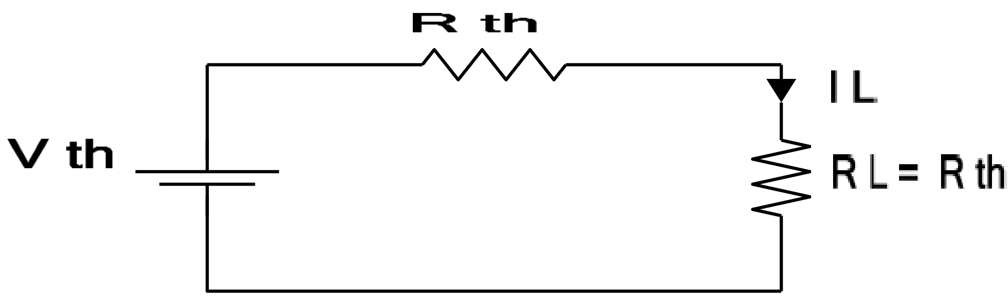
$$R_L = 2\ \Omega: \quad I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$$R_L = 10\ \Omega: \quad I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$$R_L = 100\ \Omega: \quad I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.059\ A}$$

➢ Maximum power transfer theorem :

A resistor load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals with all voltage sources removed leaving behind their internal resistances and all current sources replaced by open circuit .



Thevenins equivalent circuit

$$R_L = R_{th}$$

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$I_L = \frac{V_{th}}{2 R_{th}}$$

$$P = (I_L)^2 \times R_{th}$$

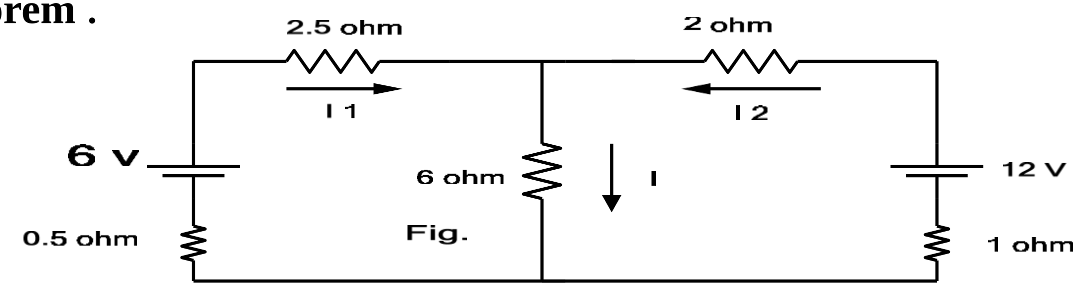
$$P_{max} = \frac{(V_{th})^2}{4 (R_{th})^2} \times R_{th}$$

$$P_{max} = \frac{(V_{th})^2}{4 R_{th}}$$

➢ Superposition theorem :

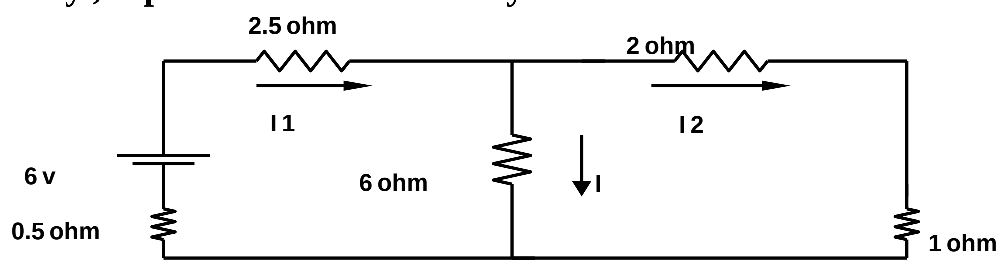
In a network containing more than one source , the current which flows at any point is the sum of all currents which would flow through that point if each source was considered separately and all the other sources replaced for the time being by resistance equal to their internal resistances .

<u>Example</u> : For the circuit shown in fig. 1 , find the current in all branches , using Superposition theorem .



1

1. Consider 6 volt only , replaced 12 volt source by its internal resistance .



$$R_t = 2.5 + \frac{3 \times 6}{3 + 6} + 0.5 = 5\ \Omega$$

$$I_1' = \frac{V_t}{R_t} = \frac{6}{5} = 1.2\ A \rightarrow$$

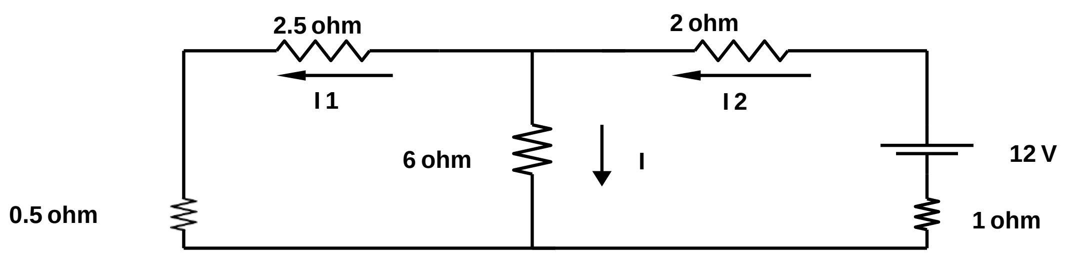
Using C.D.R :

$$I_2' = I_1' \times \frac{6}{6+3}$$

$$I_2' = 1.2 \times \frac{6}{9} = 0.8\ A \rightarrow$$

$$I' = I_1' - I_2' = 1.2 - 0.8 = 0.4\ A \downarrow$$

2. Consider 12 volt only , replaced 6 volt source by its internal resistance .



$$R_t = 2 + \frac{3 \times 6}{3 + 6} + 1 = 5\ \Omega$$

$$I_2 = \frac{V_t}{R_t} = \frac{12}{5} = 2.4\ A \leftarrow$$

Using C.D.R :

$$I_1 = I_2 \times \frac{6}{6 + 3}$$

$$I_1 = 2.4 \times \frac{6}{9} = 1.6\ A \leftarrow$$

$I = I_2 - I_1 = 2.4 – 1.6 = 0.8\ A \downarrow$

Now , take 6 volt and 12 volt sources in consideration : $I = I' + I'' = 0.4 + 0.8$

$= 1.6 – 1.2 = 0.4\ A \leftarrow$

$= 2.4 – 0.8 = 1.6\ A \leftarrow$

$= 1.2\ A \downarrow$

$I_1 = I_1'' - I_1'$ $I_2 = I_2'' - I_2'$

Example :

For the circuit shown in fig. 2 , find the current flows through 10 Ω resistor , using super position theorem .

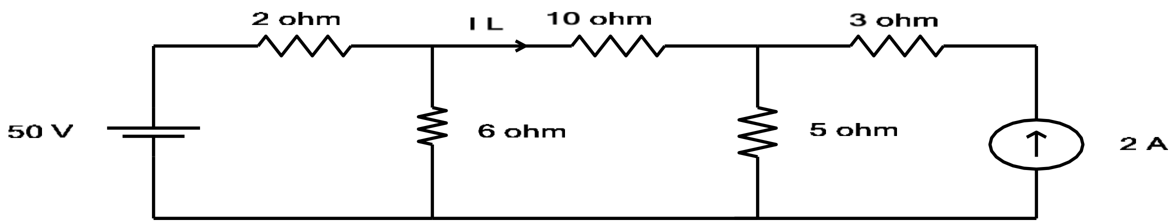
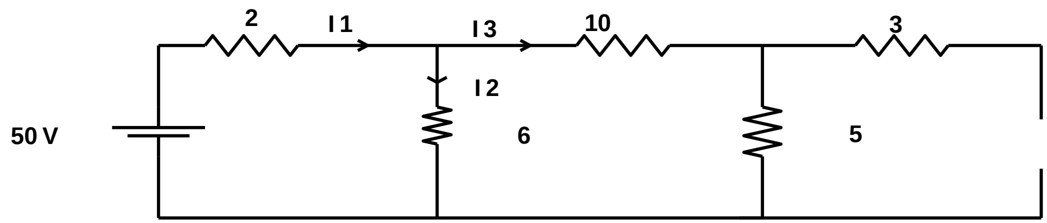


Fig. 2

1. Consider 50 volt source only , replace 2 A current source by open circuit .



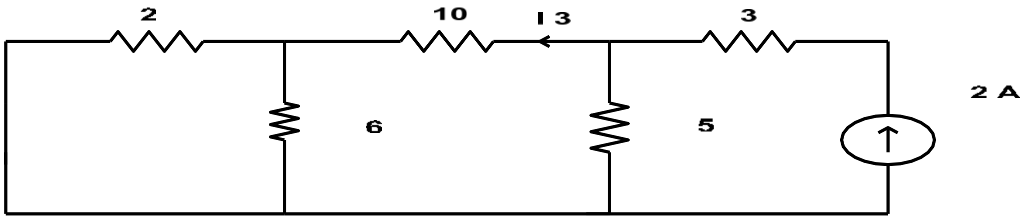
$$R_t = 2 + \frac{6 \times 15}{6 + 15} = 6.285\ \Omega$$

$$I_1 = \frac{V_t}{R_{tr}} = \frac{50}{6.285} = 7.955\ A$$

Using C.D.R :

$$I_3 = I_1 \times \frac{6}{6 + 15} = 7.955 \times \frac{6}{21} = 2.272\ A \rightarrow$$

2. Consider 2 A current source only , replaced 50 volt source by short circuit .



$$\frac{2 \times 6}{2 + 6} + 10 = 11.5\ \Omega$$

It is clear that 11.5 Ω ⁄⁄ 5 Ω

Using C.D.R :

$$I_3' = 2 \times \frac{5}{5 + 11.5} = 0.606\ A \leftarrow$$

$$I_{10\Omega} = I_3 - I_3' = 2.272 - 0.606 = 1.66\ A \rightarrow$$

RECIPROCITY THEOREM

The reciprocity theorem is applicable only to single-source networks. It is, therefore, not a theorem employed in the analysis of multisource networks described thus far. The theorem states the following:

The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

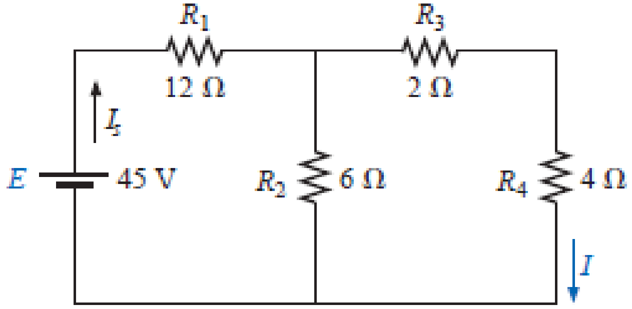
Example:

For the network shown in figure , determine the current I ,Is the reciprocity theorem satisfied?

$$R_T = R_1 + R_2 \parallel (R_3 + R_4) = 12\ \Omega + 6\ \Omega \parallel (2\ \Omega + 4\ \Omega)$$
$$= 12\ \Omega + 6\ \Omega \parallel 6\ \Omega = 12\ \Omega + 3\ \Omega = 15\ \Omega$$

and $$I_s = \frac{E}{R_T} = \frac{45\ \text{V}}{15\ \Omega} = 3\ \text{A}$$

with $$I = \frac{3\ \text{A}}{2} = 1.5\ \text{A}$$



Finding the current I due to a source E.

$$R_T = R_4 + R_3 + R_1 \parallel R_2$$
$$= 4\,\Omega + 2\,\Omega + 12\,\Omega \parallel 6\,\Omega = 10\,\Omega$$

and $$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{10\,\Omega} = 4.5\text{ A}$$

so that $$I = \frac{(6\,\Omega)(4.5\text{ A})}{12\,\Omega + 6\,\Omega} = \frac{4.5\text{ A}}{3} = 1.5\text{ A}$$