

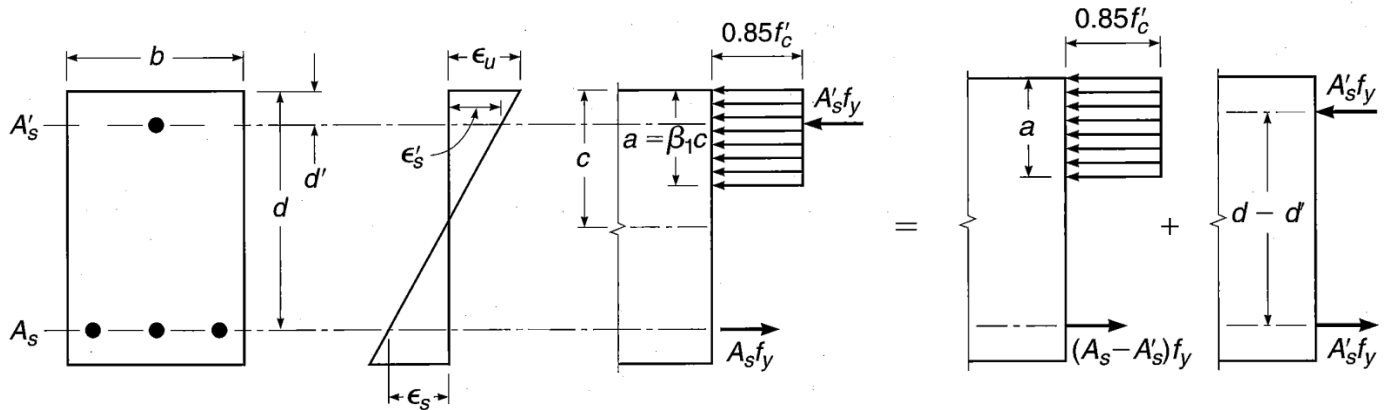
Analysis and Design of Doubly Reinforced Rectangular Beam

Analysis of Doubly Reinforced Beam

The steel that is occasionally used on the compression sides of beams is called compression steel, and beams with both tensile and compressive steel are referred to as doubly reinforced beams.

Occasionally, however, space or aesthetic requirements limit beams to such small sizes that compression steel is needed in addition to tensile steel.

- 1- To increase the moment capacity of a beam beyond that of a tensile reinforced beam with the maximum percentage of steel, it is necessary to introduce another resisting couple in the beam. This is done by adding steel in both the compression and tensile sides of the beam.
- 2- Compressive steel increases the amount of curvature that a member can take before flexural failure.
- 3- Compression steel is very effective in reducing long-term deflection due to shrinkage and plastic flow.
- 4- Continuous compression bars are also helpful for positioning stirrups (by tying them to the compression bars) and keeping them in place during concrete placement and vibration.



There are two cases

Case 1: Tension and compression steel both at yield stress

Assume $f_s = f'_s = f_y$

From $T_2 = C_2 \rightarrow A_{s2} \times f_y = A'_s \times f'_s \rightarrow A_{s2} = A'_s$

$M_{n2} = A'_s f_y (d - d')$

From $T_1 = A_{s1} \times f_y$ Where $A_{s1} = A_s - A'_s$

$M_{n1} = T_1 (d - \frac{a}{2})$

$$M_{n1} = A_{s1}f_y \left(d - \frac{a}{2}\right)$$

$$T_1 = C_1 \rightarrow A_{s1}f_y = 0.85f'_c b a \rightarrow a = \frac{A_{s1}f_y}{0.85f'_c b} \quad \text{and} \quad c = \frac{a}{\beta_1}$$

$$\text{Check } f'_s \quad f'_s = 600 \frac{c-d'}{c} \quad \text{If } f'_s \geq f_y \quad \text{the assumption is ok}$$

$$M_n = M_{n1} + M_{n2} = (A_s - A'_s)f_y \left(d - \frac{a}{2}\right) + A'_s f_y (d - d')$$

Note: Another important condition should be provided $\rho \leq \rho'_{\max} = \rho_{\max} + \rho' \frac{f'_s}{f_y}$ to ensured yielding of tension steel

Case 2: Compression steel below yield stress

If $f'_s < f_y$ and $\rho \leq \rho'_{\max}$ but $(f_s = f_y)$

$$T = C_1 + C_2 \rightarrow A_s f_y = 0.85f'_c b a + A'_s f'_s$$

$$\text{From } a = \beta_1 c \text{ and } f'_s = 600 \frac{c-d'}{c}$$

$$A_s f_y = 0.85f'_c b \beta_1 c + A'_s 600 \frac{c-d'}{c}$$

Solved equation to find c and then find a & f'_s

$$\text{Find } A_{s2} = A'_s \frac{f'_s}{f_y} \quad \text{and} \quad A_{s1} = A_s - A_{s2}$$

$$M_n = M_{n1} + M_{n2} = A_{s1}f_y \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')$$

$$\text{Or } M_n = 0.85f'_c b a \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d') \quad \text{and} \quad M_u = \phi M_n$$

$$\text{Check } \rho_{act} = \frac{A_s}{bd} \rightarrow \rho_{min} \leq \rho_{act} \leq \rho'_{\max}$$

$$\text{Where } \rho_{min} = \frac{\sqrt{f'_c}}{4f_y} \geq \frac{1.4}{f_y}$$

$$\rho'_{\max} = \rho_{\max} + \rho' \frac{f'_s}{f_y}$$

$$\rho'_{\max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \rho' \frac{f'_s}{f_y} \quad \text{where } \rho' = \frac{A'_s}{bd}$$

Ex1: Find the ultimate bending moment for the section shown in figure below, $f'_c = 20.7\text{MPa}$, $f_y = 400\text{MPa}$.

Check the reason for using of compression reinforcement:

$$A_s = 5 \times \pi \left(\frac{32}{2}\right)^2 = 4021.24 \text{ mm}^2$$

$$\rho_{provided} = \frac{A_s}{bd} = \frac{4021.24}{360 \times 600} = 0.0186$$

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.016 < \rho_{provided}$$

Compression reinforcement has been added for change mode from compression failure to tension failure.

Assume $f'_s = f_y$

$$A'_s = 2 \times \pi \left(\frac{25}{2}\right)^2 = 981.75 \text{ mm}^2 = A_{s2}$$

$$A_{s1} = 4021.24 - 981.75 = 3039.5 \text{ mm}^2$$

$$a = \frac{A_{s1}f_y}{0.85f'_c b} = \frac{3039.5 \times 400}{0.85 \times 20.7 \times 360} = 191.94 \text{ mm} \quad \text{And} \quad c = \frac{a}{\beta_1} = \frac{191.94}{0.85} = 225.8 \text{ mm}$$

$$f'_s = 600 \frac{c - d'}{c} = 600 \times \frac{225.8 - 60}{225.8} = 440 \text{ MPa} \geq 400 \text{ MPa} \quad \text{ok}$$

$$\rho_{min} = \frac{\sqrt{f'_c}}{4f_y} = \frac{\sqrt{20.7}}{4 \times 400} = 0.0028 \geq \frac{1.4}{f_y} = \frac{1.4}{400} = 0.0035 \quad \text{control}$$

$$\rho_{act} = \frac{A_s}{bd} = \frac{4021.24}{360 \times 600} = 0.0186 < \rho'_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \rho'$$

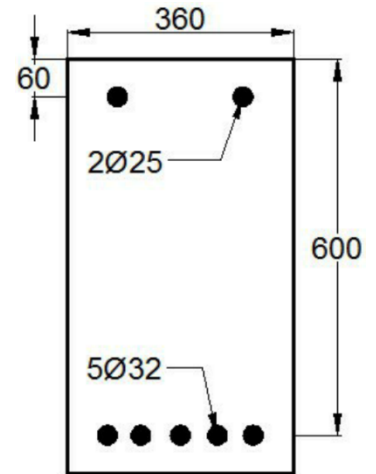
$$= 0.85 \times 0.85 \times \frac{20.7}{400} \times \frac{0.003}{0.003 + 0.004} + \frac{981.75}{360 \times 600} = 0.0205 \quad \text{ok}$$

$$\epsilon_t = \frac{d - c}{c} \times 0.003 = 0.005 \quad \text{use } \phi = 0.9$$

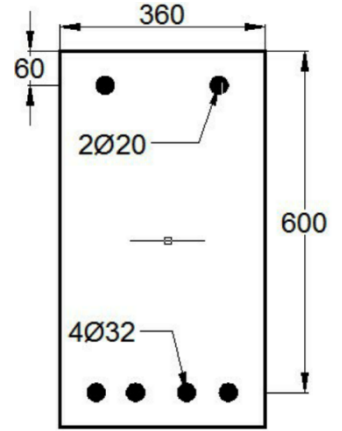
$$M_u = \phi [A_{s1}f_y \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')]]$$

$$M_u = 0.9 \left[3039.5 \times 400 \left(600 - \frac{191.94}{2}\right) + 981.75 \times 400 (600 - 60) \right] \times 10^{-6}$$

$$M_u = 742.37 \text{ kN.m}$$



Ex2: Find the ultimate bending strength for the doubly reinforced concrete section shown in figure below, use $f'_c = 27.6MPa$, $f_y = 345MPa$



$$A_s = 4 \times \pi \left(\frac{32}{2} \right)^2 = 3217 mm^2$$

$$A'_s = 2 \times \pi \left(\frac{20}{2} \right)^2 = 628 mm^2$$

Assume $f'_s = f_y$ and $A'_s = A_{s2}$

$$A_{s1} = A_s - A'_s = 3217 - 628 = 2589 mm^2$$

$$a = \frac{A_{s1} f_y}{0.85 f'_c b} = \frac{2589 \times 345}{0.85 \times 27.6 \times 360} = 105.7 mm \quad \text{And} \quad c = \frac{a}{\beta_1} = \frac{105.7}{0.85} = 124.3 mm$$

$$\text{Check } f'_s = 600 \frac{c-d'}{c} = 600 \times \frac{124.3-60}{124.3} = 310 MPa < 345 MPa \quad \text{the Assumption not ok}$$

$$T = C_1 + C_2 \rightarrow A_s f_y = 0.85 f'_c b a + A'_s f'_s \quad a = \beta_1 c \quad \text{and} \quad f'_s = 600 \frac{c-d'}{c}$$

$$A_s f_y = 0.85 f'_c b \beta_1 c + A'_s 600 \frac{c-d'}{c}$$

$$3217 \times 345 = 0.85 \times 27.6 \times 360 \times 0.85 c + 628 \times 600 \times \frac{c-60}{c}$$

$$c^2 - 102.12c - 3149.29 = 0 \rightarrow c = 126.93 mm \quad a = \beta_1 c = 0.85 \times 126.93 = 107.9 mm$$

$$f'_s = 600 \frac{c-d'}{c} = 600 \frac{126.93-60}{126.93} = 316.38 MPa < 345 MPa$$

$$\text{Check } A_{s2} = A'_s \frac{f'_s}{f_y} = 628 \times \frac{316.38}{345} = 575.9 mm^2$$

$$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} = \frac{\sqrt{27.6}}{4 \times 345} = 0.0038 \geq \frac{1.4}{f_y} = \frac{1.4}{345} = 0.004 \quad (\text{Control})$$

$$\rho_{act} = \frac{A_s}{bd} = \frac{3217}{360 \times 600} = 0.01489 \quad \rho'_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \rho' \frac{f'_s}{f_y}$$

$$\rho'_{max} = 0.85 \times 0.85 \times \frac{20.7}{345} \times \frac{0.003}{0.003+0.004} + \frac{628}{360 \times 600} \frac{316.38}{345} = 0.0384$$

$$\rho_{min} = 0.004 \leq \rho_{act} = 0.01489 \leq \rho'_{max} = 0.0384$$

$$\text{Calculate } \phi \quad \epsilon_s = \epsilon_t = 0.003 \frac{d-c}{c} = 0.003 \times \frac{600-126.93}{126.93} = 0.011 \geq 0.005 \quad \text{use } \phi = 0.9$$

$$M_u = \phi \left[0.85 f'_c b a \left(d - \frac{a}{2} \right) + A'_s f'_s (d - d') \right] = 0.9 \left[0.85 \times 27.6 \times 360 \times 107.9 \times \left(600 - \frac{107.9}{2} \right) + 628 \times 316.38 (600 - 60) \right] \times 10^{-6} = 544.4 kN.m$$

Design of Doubly reinforced concrete Rectangular beams

Find the ultimate applied moment M_u , external moment

Calculate $\rho_{\max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}$ to Find $A_{s\max} = \rho_{\max} bd$

Calculate $a = \frac{A_{s\max} f_y}{0.85 f'_c b}$ and $c = \frac{a}{\beta_1}$

Use $\phi = 0.816$ calculate from $\phi = 0.65 + (\varepsilon_t - 0.002) \frac{250}{3}$

$$\phi = 0.65 + (0.004 - 0.002) \frac{250}{3} = 0.8167$$

Calculate internal moment $\phi M_n = \phi A_{s\max} f_y (d - \frac{a}{2})$

Compare M_u with ϕM_n

1- If $M_u > \phi M_n$ Thus section is doubly reinforced

So that $A_{s1} = A_{s\max}$, $M_{u1} = \phi M_n \rightarrow M_{u2} = M_u - M_{u1}$

$$A_{s2} = \frac{M_{u2}}{\phi f_y (d - d')}$$

$$f'_s = 600 \frac{c - d'}{c}$$

If $f'_s \geq f_y$ so $A'_s = A_{s2}$ and $A_s = A_{s1} + A_{s2}$

If $f'_s < f_y$ so $A'_s = A_{s2} \frac{f_y}{f'_s}$ and $A_s = A_{s1} + A_{s2}$

2- If $M_u \leq \phi M_n$ Thus section is singly reinforced

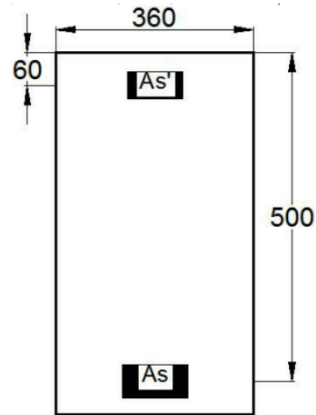
So find ρ_{act} , from $M_u = \phi \rho f_y b d^2 (1 - 0.59 \rho \frac{f_y}{f'_c}) \rightarrow \rho = \frac{(1 - \sqrt{1 - \frac{2.36 M_u}{\phi b d^2 f'_c}})}{(1.18 \frac{f_y}{f'_c})}$

Check $\rho_{\min} \leq \rho_{act, req} \leq \rho_{\max}$

$$\rho_{\min} = \frac{\sqrt{f'_c}}{4 f_y} \geq \frac{1.4}{f_y}$$

$$A_s = \rho_{act} b d$$

Ex3: Find the steel reinforcement area for the section shown in figure below, Assume M.D.L = 150kN.m, M.L.L.= 200kN.m, $f'_c = 20.7\text{MPa}$, $f_y = 414\text{MPa}$.



$$M_{u\text{ ext}} = 1.2 \times 150 + 1.6 \times 200 = 500\text{kN.m}$$

$$\rho_{\text{max}} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \times \frac{20.7}{414} \frac{0.003}{0.003 + 0.004} = 0.0155$$

$$A_{s\text{ max}} = \rho_{\text{max}} bd = 0.0155 \times 360 \times 500 = 2786.79\text{mm}^2$$

$$a = \frac{A_{s\text{ max}} f_y}{0.85 f'_c b} = \frac{2786.79 \times 414}{0.85 \times 20.7 \times 360} = 182.14\text{mm} \quad c = \frac{a}{\beta_1} = \frac{182.14}{0.85} = 214.28\text{mm}$$

$$\phi = 0.65 + (\epsilon_t - 0.002) \frac{250}{3} = 0.65 + (0.004 - 0.002) \frac{250}{3} = 0.8167$$

$$\phi M_n = \phi A_{s\text{ max}} f_y \left(d - \frac{a}{2}\right) = 0.816 \times 2786.79 \times 414 \times \left(500 - \frac{182.14}{2}\right) \times 10^{-6} = 385\text{kN.m}$$

$M_{u\text{ ext}} = 500 > \phi M_n = 385$ So the section should be design as doubly reinforced

$$A_{s1} = A_{s\text{ max}} = 2786.79\text{mm}^2$$

$$M_{u1} = \phi M_n = 385\text{kN.m}, M_{u2} = M_u - M_{u1} = 500 - 385 = 115\text{kN.m}$$

$$A_{s2} = \frac{M_{u2}}{\phi f_y (d - d')} = \frac{115 \times 10^6}{0.816 \times 414 \times (500 - 60)} = 733.68\text{mm}^2$$

$$f'_s = 600 \frac{c - d'}{c} = 600 \times \frac{214.28 - 60}{214.28} = 432\text{MPa} > f_y = 414\text{MPa}$$

$$A'_s = A_{s2} = 733.68\text{mm}^2 \text{ And } A_s = A_{s1} + A_{s2} = 2786.79 + 733.68 = 3560.5\text{mm}^2$$

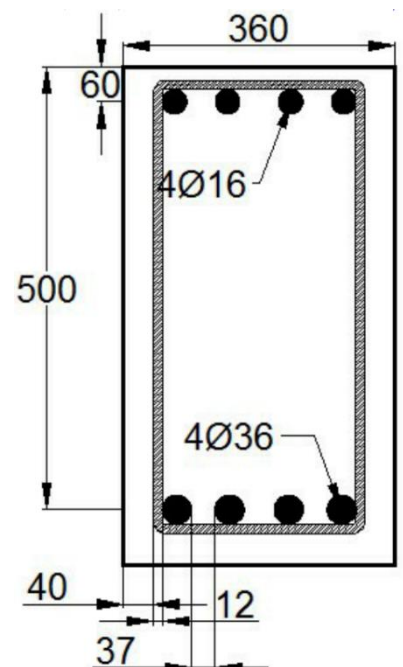
$$\text{Assume } \phi = 35\text{mm No. of bars} = \frac{3560.5}{18^2 \pi} = 3.45 \approx 4\phi 36$$

$$S = \frac{360 - 2 \times 40 - 2 \times 12.5 - 4 \times 36}{3} = 37 \geq \begin{cases} 25\text{ mm} \\ Db = 36\text{ mm} \\ \frac{4}{3} \text{ M.A.S.} \end{cases}$$

For compression reinforcement, Assume $\phi = 16\text{mm}$,

$$A_b = 201$$

$$\text{No.} = \frac{733.68}{201} = 3.65 \approx 4 \text{ Use } 4\phi 16$$



Ex4: For cantilever beam $L = 4m$ find the steel reinforcement area, if the live load is 20 kN/m and the dead load within self weight for the beam is 18 kN/m where $f'_c = 21 \text{ MPa}$, $f_y = 400 \text{ MPa}$, $d = 510 \text{ mm}$, $d' = 90 \text{ mm}$ $b = 300 \text{ mm}$.

$$W_u = 1.2 \times 18 + 1.6 \times 20 = 53.6 \text{ kN/m}$$

$$M_{u \text{ ext}} = \frac{W_u L^2}{2} = \frac{53.6 \times 4^2}{2} = 428.8 \text{ kN.m}$$

$$\rho_{\max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85 \times 0.85 \times \frac{21}{400} \frac{0.003}{0.003 + 0.004} = 0.01626$$

$$A_{s \max} = \rho_{\max} b d = 0.01626 \times 300 \times 510 = 2487.78 \text{ mm}^2$$

$$a = \frac{A_{s \max} f_y}{0.85 f'_c b} = \frac{2487.78 \times 400}{0.85 \times 21 \times 300} = 185.83 \text{ mm} \quad c = \frac{a}{\beta_1} = \frac{185.83}{0.85} = 218.62 \text{ mm}$$

$$\phi = 0.65 + (\epsilon_t - 0.002) \frac{250}{3} = 0.65 + (0.004 - 0.002) \frac{250}{3} = 0.8167$$

$$\phi M_n = \phi A_{s \max} f_y \left(d - \frac{a}{2}\right) = 0.816 \times 2487.78 \times 400 \times \left(510 - \frac{185.83}{2}\right) \times 10^{-6} = 338.45 \text{ kN.m}$$

$$\text{Or } \phi M_n = \phi \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f_y}{f'_c}\right) = 0.816 \times 0.01626 \times 400 \times 300 \times 510^2 \left(1 - 0.59 \times 0.01626 \times \frac{400}{21}\right) \times 10^{-6} = 338.45 \text{ kN.m} < 428.8 \text{ so the section is doubly reinforcement}$$

$$A_{s1} = A_{s \max} = 2487.78 \text{ mm}^2$$

$$M_{u1} = \phi M_n = 338.45 \text{ kN.m}, \quad M_{u2} = M_u - M_{u1} = 428.8 - 338.45 = 90.35 \text{ kN.m}$$

$$A_{s2} = \frac{M_{u2}}{\phi f_y (d - d')} = \frac{90.35 \times 10^6}{0.816 \times 400 \times (510 - 90)} = 658.5 \text{ mm}^2$$

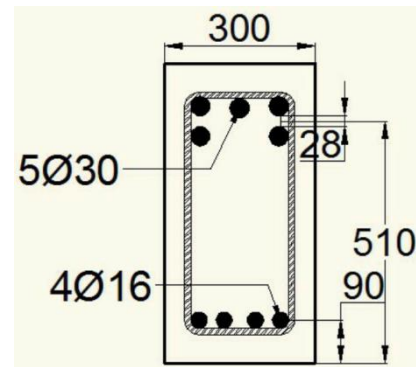
$$f'_s = 600 \frac{c - d'}{c} = 600 \times \frac{218.62 - 90}{218.62} = 353 \text{ MPa} < f_y = 400 \text{ MPa}$$

$$A'_s = A_{s2} \frac{f_y}{f'_s} = 658.5 \times \frac{400}{353} = 746.18 \text{ mm}^2$$

$$A_s = A_{s1} + A_{s2} = 2478.78 + 658.5 = 3137.28 \text{ mm}^2$$

$$\text{Use } \phi = 30 \text{ mm No.} = \frac{3137.28}{707} = 4.44 \approx 5 \text{ use } 5\phi 30$$

$$\text{Use } \phi = 16 \text{ mm No.} = \frac{746.18}{201} = 3.71 \approx 4 \text{ } 4\phi 16$$

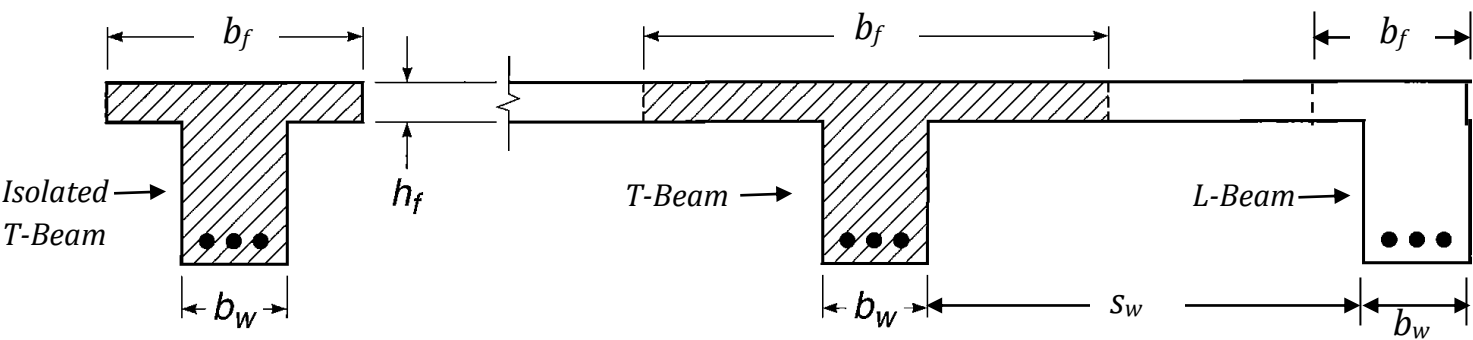


$$\rho_{\min} = 0.0035 \leq \rho_{\text{req}} = \frac{3137.28}{300 \times 510} = 0.0205 \leq \rho'_{\max} = 0.01625 + \frac{746.18}{300 \times 510} = 0.211$$

Analysis and Design of T-section Beam

Reinforced concrete floors are almost always monolithic. A form are built for beam soffits and sides and for the under-side of slabs, and the entire construction is cast at once, from the bottom of the deepest beam to the top of the slab. Beam stirrups and bent bars extend up into the slab. It is evident, therefore, that a part of the slab will act with the upper part of the beam to resist longitudinal compression. The resulting beam cross section is T-shaped rather than rectangular. The slab forms the beam flange, while the part of the beam projecting below the slab forms what is called the *web* or *stem*.

T-beam geometry



For Isolated nonprestressed *T*-beams in which the flange is used to provide additional compression area shall have a flange thickness greater than or equal to $0.5b_w$ and an effective flange width less than or equal to $4b_w$.

$h_f \geq 0.5b_w$ and $b_f \leq 4b_w$

-For nonprestressed *T*-beams supporting monolithic or composite slabs, the effective flange width b_f shall include the beam web width b_w plus an effective overhanging flange width in accordance with the Table below, where h_f is the slab thickness and s_w is the clear distance to the adjacent web.

Flange location	Effective overhanging flange width, beyond face of web	
Each side of web	Least of:	$8h_f$
		$s_w/2$
		$\ell_n/8$
One side of web	Least of:	$6h_f$
		$s_w/2$
		$\ell_n/12$

For symmetric *T* beams, the effective width b_f shall not exceed one-fourth the span length of the beam. The overhanging slab width on either side of the beam web shall not exceed 8 times the thickness of the slab or go beyond one-half the clear distance to the next beam.

$$b_f = b_w + 2 \times \text{the Least value of table} \rightarrow b_f = \min[b_w + 16h_f, \quad b_w + s_w, \quad b_w + \frac{l_n}{4}]$$

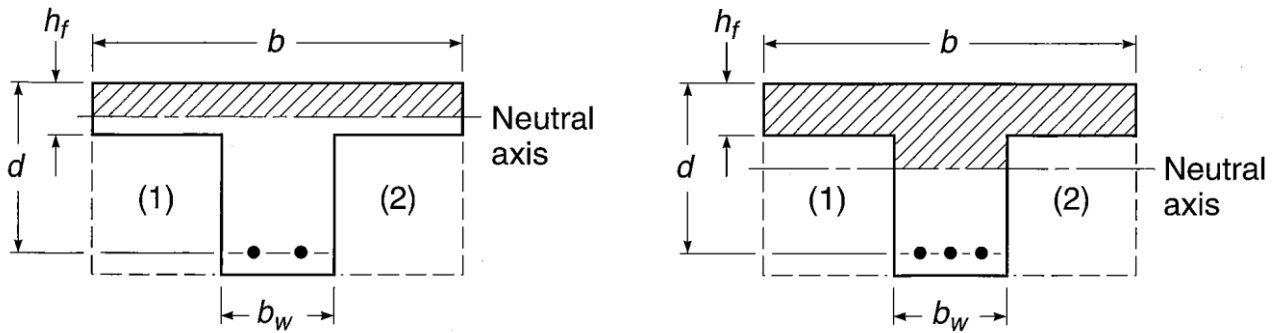
For beams having a slab on one side only (*L-Beam*), the effective overhanging slab width shall not exceed one-twelfth the span length of the beam, 6 times the slab thickness, or one-half the clear distance to the next beam.

$$b_f = b_w + \text{the Least value of table} \rightarrow b_f = \min[b_w + 6h_f, \quad b_w + \frac{s_w}{2}, \quad b_w + \frac{l_n}{12}]$$

Strength Analysis

The neutral axis of a *T* beam may be either in the flange or in the web, depending upon the proportions of the cross section, the amount of tensile steel, and the strengths of the materials.

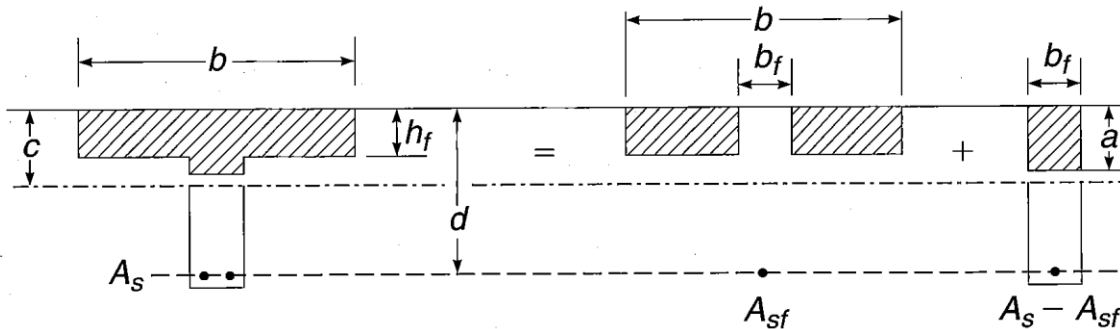
If the calculated depth to the neutral axis is less than or equal to the flange thickness h_f , the beam can be analyzed as if it were a rectangular beam of width equal to b , the effective flange width. While when the neutral axis is located in the web. In this case, methods must be developed to account for the actual *T*-shaped compressive zone.



Case 1, If $a \leq h_f$ analysis as rectangular section of width b

Case 2, If $a > h_f$ analysis as a *T* —section beam

It is convenient to divide the total tensile steel into two parts, as shown in Figure below. The first part, A_{sf} , represents the steel area that, when stressed to f_y , is required to balance the longitudinal compressive force in the over-hanging portions of the flange that are stressed uniformly at $0.85f'_c$. Thus,



$$A_{sf}f_y = 0.85f'_c(b - b_w)h_f \quad \rightarrow \quad A_{sf} = \frac{0.85f'_c(b - b_w)h_f}{f_y}$$

$$M_{nf} = A_{sf} f_y (d - \frac{h_f}{2})$$

$$(A_s - A_{sf}) f_y = 0.85 f'_c a b_w \rightarrow a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w}$$

$$M_{nw} = (A_s - A_{sf}) f_y (d - \frac{a}{2})$$

$$M_n = M_{nf} + M_{nw} = A_{sf} f_y (d - \frac{h_f}{2}) + (A_s - A_{sf}) f_y (d - \frac{a}{2})$$

Analysis of T-section Beam

- Check the beam dimensions
- Check the section (Rectangular or T- section)

$$a = \frac{A_{sf} f_y}{0.85 f'_c b_f} \text{ and compare}$$

If $a \leq h_f$ Rectangular section

If $a > h_f$ T- section

- Calculate $A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y}$
- Calculate $\rho_w = \frac{A_s}{b_w d}$ $\rho_f = \frac{A_{sf}}{b_w d}$ $\rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{600}{600 + f_y}$
- Compare ρ_w with ρ_{wb}

If $\rho_w \leq \rho_{wb} \rightarrow$ under reinforced section $f_s = f_y$

$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w} \rightarrow M_n = A_{sf} f_y (d - \frac{h_f}{2}) + (A_s - A_{sf}) f_y (d - \frac{a}{2})$$

If $\rho_w > \rho_{wb} \rightarrow$ over reinforced section $f_s < f_y$

Find c and f_s

$$f_s = 600 \frac{d - c}{c}$$

$$A_s f_s = 0.85 f'_c (b - b_w) h_f + 0.85 f'_c a b_w$$

$$A_s (600 \frac{d - c}{c}) = 0.85 f'_c (b - b_w) h_f + 0.85 f'_c \beta_1 c b_w \rightarrow c \text{ and } a = \beta_1 c$$

$$\text{Calculate } M_n = 0.85 f'_c (b - b_w) h_f (d - \frac{h_f}{2}) + 0.85 f'_c a b_w (d - \frac{a}{2})$$

Find ϕ and Calculate $M_u = \phi M_n$

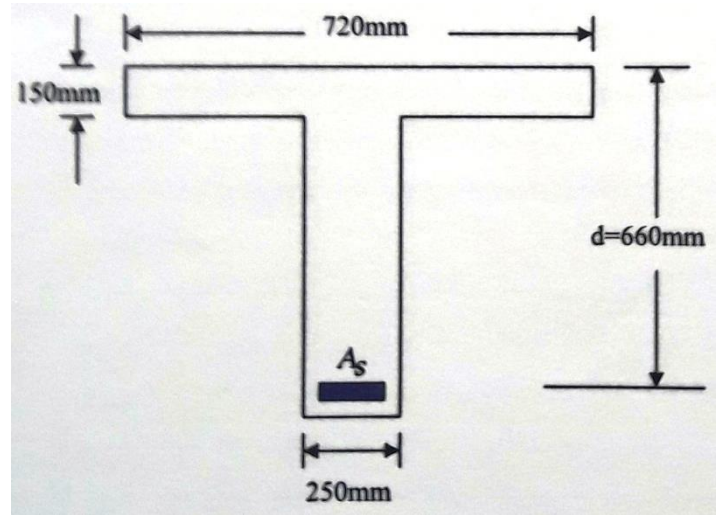
$$\rho_{min} \leq \rho_{w \text{ act}} \leq \rho_{w \text{ max}} \text{ Where } \rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} \geq \frac{1.4}{f_y}$$

Ex5: For the following T- reinforced concrete beam with ($b_f = 720mm$), ($d = 660mm$), ($h_f = 150mm$), ($b_w = 250mm$), $f'_c = 20MPa$ and $f_y = 420MPa$, calculate the design moment when (1) $A_s = 3000mm^2$ and (2) $A_s = 4800mm^2$ and (3) $A_s = 7000mm^2$.

Check the beam dimensions

$$h_f = 150mm \geq 0.5b_w = 125mm$$

$$b_f = 720mm \leq 4b_w = 1000mm \quad \text{ok}$$



$$(1) A_s = 3000mm^2$$

Check the section (Rectangular or T- section)

$$a = \frac{A_s f_y}{0.85 f'_c b_f} = \frac{3000 \times 420}{0.85 \times 20 \times 720} = 103mm \leq h_f = 150mm$$

Rectangular section with (720×660mm)

$$\rho = \frac{A_s}{bd} = \frac{3000}{720 \times 660} = 0.0063$$

$$\rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{600}{600 + f_y} = 0.85^2 \frac{20}{400} \frac{600}{600 + 400} = 0.0202 > \rho = 0.0063 \quad \text{under reinforced section}$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = 3000 \times 420 \times \left(660 - \frac{103}{2} \right) \times 10^{-6} = 766.7 kN.m$$

$$a = \beta_1 c \rightarrow 103 = 0.85 \times c \rightarrow c = 121mm$$

$$\varepsilon_{t=} \frac{d-c}{c} \varepsilon_{cu} = \frac{660-121}{121} \times 0.003 = 0.0133 > 0.005, \text{ So } \phi = 0.9$$

$$M_u = \phi M_n = 0.9 \times 766.7 = 690 kN.m$$

$$(2) = 4800mm^2$$

Check the section (Rectangular or T- section)

$$a = \frac{A_s f_y}{0.85 f'_c b_f} = \frac{4800 \times 420}{0.85 \times 20 \times 720} = 164.7mm > h_f = 150mm \quad \text{T- section}$$

$$A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y} = \frac{0.85 \times 20 \times (720 - 250) \times 150}{420} = 2854mm^2$$

$$\rho_f = \frac{A_{sf}}{b_w d} = \frac{2854}{250 \times 660} = 0.0173$$

$$\rho_{wb} = \rho_b + \rho_f = 0.0202 + 0.0173 = 0.0375$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{4800}{250 \times 660} = 0.029 \leq \rho_{wb} = 0.0375 \text{ Under reinforced section } f_s = f_y$$

$$a = \frac{(A_s - A_{sf})f_y}{0.85f'_c b_w} = \frac{(4800 - 2854) \times 420}{0.85 \times 20 \times 250} = 192.3 \text{ mm}$$

$$M_n = A_{sf}f_y(d - \frac{h_f}{2}) + (A_s - A_{sf})f_y(d - \frac{a}{2})$$

$$M_n = [2854 \times 420 \times (660 - \frac{150}{2}) + (4800 - 2854) \times 420 (660 - \frac{192.3}{2})] \times 10^{-6}$$

$$M_n = 1162 \text{ kN.m}$$

$$a = \beta_1 c \rightarrow 192.3 = 0.85 \times c \rightarrow c = 226.23 \text{ mm}$$

$$\varepsilon_t = \frac{d-c}{c} \varepsilon_{cu} = \frac{660 - 226.23}{226.23} \times 0.003 = 0.00575 > 0.005, \text{ So } \phi = 0.9$$

$$M_u = \phi M_n = 0.9 \times 1162 = 1045.8 \text{ kN.m}$$

$$(3) = 7000 \text{ mm}^2$$

Check the section (Rectangular or T- section)

$$a = \frac{A_s f_y}{0.85 f'_c b_f} = \frac{7000 \times 420}{0.85 \times 20 \times 720} = 240 \text{ mm} > h_f = 150 \text{ mm} \quad \text{T- section}$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{7000}{250 \times 660} = 0.042 \leq \rho_{wb} = 0.0375 \text{ Over reinforced section } f_s < f_y$$

Find c and f_s

$$A_s(600 \frac{d-c}{c}) = 0.85 f'_c (b - b_w) h_f + 0.85 f'_c \beta_1 c b_w$$

$$7000(600 \times \frac{660 - c}{c}) = 0.85 \times 20(720 - 250) \times 150 + 0.85^2 \times 20 \times c \times 250$$

$$\rightarrow c = 404 \text{ mm} \quad \text{and} \quad a = \beta_1 c = 0.85 \times 404 = 343 \text{ mm}$$

$$f_s = 600 \frac{d-c}{c} = 600 \times \frac{660 - 404}{404} = 380 \text{ MPa} < f_y = 420 \text{ MPa}$$

$$M_n = 0.85 f'_c (b - b_w) h_f (d - \frac{h_f}{2}) + 0.85 f'_c a b_w (d - \frac{a}{2})$$

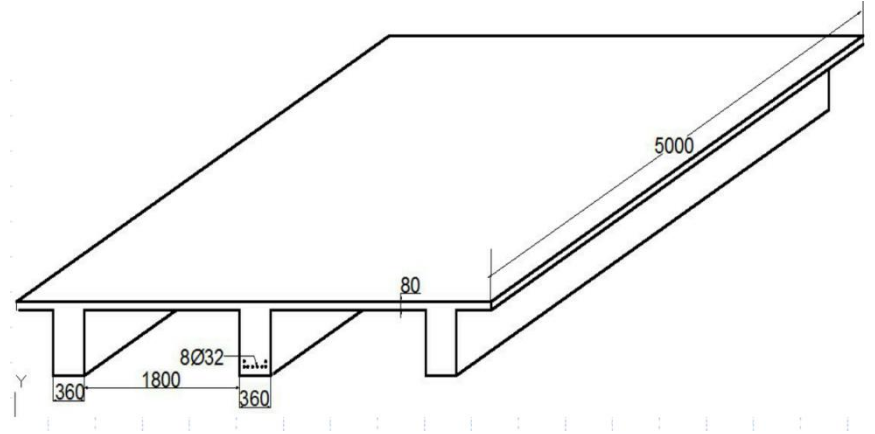
$$M_n = \left[0.85 \times 20 \times (720 - 250) \times 150 \times \left(660 - \frac{150}{2} \right) + 0.85 \times 20 \times 343 \times 250 \right. \\ \left. \times \left(600 - \frac{343}{2} \right) \right] \times 10^{-6} = 1413.23 \text{ kN.m}$$

$$\varepsilon_t = \frac{d-c}{c} \varepsilon_{cu} = \frac{660 - 404}{404} \times 0.003 = 0.0019 < 0.002, \text{ So } \phi = 0.65$$

$$M_u = \phi M_n = 0.65 \times 1413.23 = 918.6 \text{ kN.m}$$

Ex6: For the slab-beam system shown in Figure below, Find the ultimate bending moment can be carried when $f'_c = 20.7\text{MPa}$ and $f_y = 345\text{MPa}$, span of beam = 5m , $d = 600\text{ mm}$, $A_s = 8\phi 32\text{mm}$.

Check the beam dimensions



$$b_f = \min \begin{cases} b_w + 16h_f = 360 + 16 \times 80 = 1640\text{mm} \\ b_w + s_w = 360 + 1800 = 2160\text{mm} \\ b_w + \frac{l_n}{4} = 360 + \frac{5000}{4} = 1610\text{mm (control)} \end{cases}$$

$$A_s = 8 \times \pi \times \left(\frac{32}{2}\right)^2 = 6434\text{mm}^2$$

Check the section (Rectangular or T- section

$$a = \frac{A_s f_y}{0.85 f'_c b_f} = \frac{6434 \times 345}{0.85 \times 20.7 \times 1610} = 78.36\text{mm} \leq h_f = 80\text{mm}$$

Rectangular section with (1610×600mm)

$$\rho = \frac{A_s}{bd} = \frac{6434}{1610 \times 600} = 0.0067$$

$$\rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{600}{600 + f_y} = 0.85^2 \frac{20.7}{400} \frac{600}{600 + 400} = 0.0224 > \rho = 0.0067 \text{ under reinforced section}$$

$$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} = \frac{\sqrt{20.7}}{4 \times 345} = 0.0033 \geq \frac{1.4}{f_y} = \frac{1.4}{345} = 0.00405$$

$$\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.85^2 \frac{20.7}{400} \frac{0.003}{0.003 + 0.004} = 0.0160$$

$$\rho_{min} \leq \rho_{act} \leq \rho_{max}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 6434 \times 345 \times \left(600 - \frac{78.36}{2}\right) \times 10^{-6} = 1244.86\text{kN.m}$$

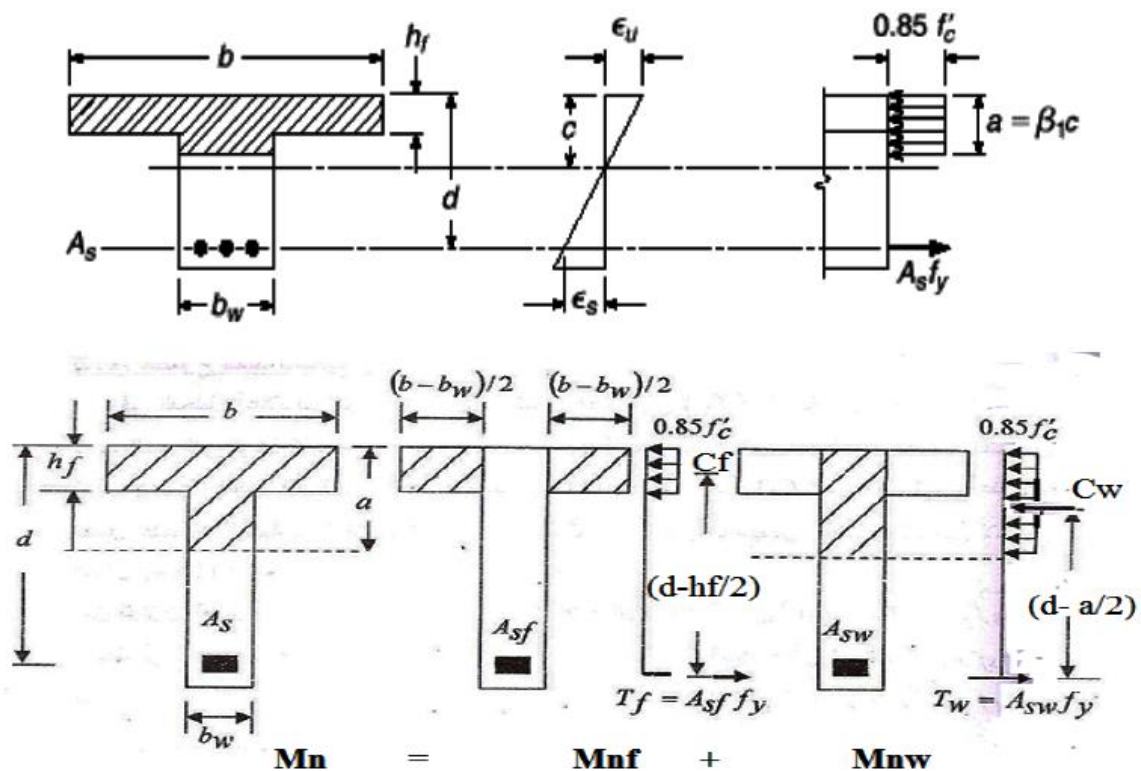
$$a = \beta_1 c \rightarrow 78.36 = 0.85 \times c \rightarrow c = 92.2\text{mm}$$

$$\epsilon_t = \frac{d-c}{c} \epsilon_{cu} = \frac{600-92.2}{92.2} \times 0.003 = 0.0165 > 0.005, \text{ So } \phi = 0.9$$

$$M_u = \phi M_n = 0.9 \times 1244.86 = 1120.38\text{kN.m}$$

Design of T-section Beam

For the design of T or L section beams the flange has normally already been selected in the slab design as it is for the slab. The size of the web is normally not selected on the basis of the moment requirements but probably is given an area based on shear requirements. That is a sufficient area is used so as to provide a certain minimum shear capacity. It is also possible that the width of the web may be selected on the basis of the width estimated to be needed to put the reinforcing bars. Size may also have been preselected to simplify formwork for architectural requirements or for deflection reasons.



- Check the beam dimensions
- Assume the section is analysed as rectangular, $a = h_f$
- Calculate $M_{u)ext}$ and assume $\phi = 0.9$

$$M_{uf} = \phi [0.85 f'_c b_f h_f (d - \frac{h_f}{2})]$$

- Compare a calculated $M_{u)ext}$ with M_{uf}

If $M_{u)ext} \leq M_{uf}$ design as Rectangular section

If $M_{u)ext} > M_{uf}$ design as T- section

- Calculate $A_{sf} = \frac{0.85 f'_c (b_f - b_w) h_f}{f_y}$

$$M_{uf} = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right)$$

$$M_{uw} = M_{u)ext} - M_{uf}$$

From M_{uw} calculate ρ and A_{sw}

Where $R = \frac{M_{uw}}{\phi b_w d^2}$, $m = \frac{f_y}{0.85 f'_c}$ and assume $\phi = 0.9$

$$\rho = \frac{1}{m} \left(1 - \sqrt{1 - \frac{2mR}{f_y}}\right) \rightarrow A_{sw} = \rho b_w d$$

- Calculate $A_s = A_{sf} + A_{sw}$
- Calculate $\rho_w, \rho_f, \rho_{w)max}$ and ρ_{min}

$$\rho_w = \frac{A_s}{b_w d} \quad \rho_f = \frac{A_{sf}}{b_w d} \quad \rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_{uc}}{\epsilon_{uc} + 0.004}$$

$$\rho_{w)max} = \rho_{max} + \rho_f$$

If $\rho_w \leq \rho_{w)max}$ section satisfy the requirement of maximum steel reinforcement ok

If $\rho_w > \rho_{w)max}$ no governing

Option for solution:

- a- Increase flange thickness (h_f)
- b- Use compression steel reinforcement (doubly reinforced section)
- c- Increase the effective depth (d)

$$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} \geq \frac{1.4}{f_y} \quad \text{If } \rho_w < \rho_{min} \text{ use } \rho_w = \rho_{min}$$

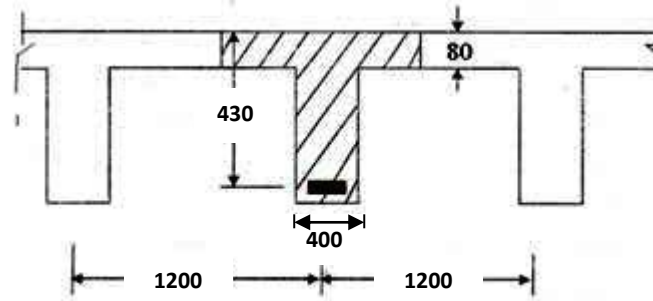
- Check ϕ

$$a = \frac{A_{sw} f_y}{0.85 f'_c b_w} \quad c = \frac{a}{\beta_1} \quad \text{and} \quad \epsilon_t = \frac{d-c}{c} \epsilon_{cu}$$

- Assume ϕb to find the number the bars $\text{No. of bars} = \frac{A_s}{A_b}$
- Check the space between the bars

$$S = \frac{b - 2cover - 2\phi stirrup - n\phi b}{n - 1} \geq \begin{cases} 25 \text{ mm} \\ Db = 36 \text{ mm} \\ \frac{4}{3} M.A.S. \end{cases}$$

Ex7: for the T-section in figure below, find the reinforcement area if $f'_c = 20.7\text{MPa}$ and $f_y = 400\text{MPa}$, span of beam = 6m when (a) $M_u = 500\text{kN.m}$, (b) $M_u = 650\text{kN.m}$, (c) $M_u = 800\text{kN.m}$,



Solution

Check the beam dimensions

$$b_f = \min \begin{cases} b_w + 16h_f = 400 + 16 \times 80 = 1680\text{mm} \\ b_w + s_w = 400 + 800 = 1200\text{mm} \text{ (control)} \\ b_w + \frac{l_n}{4} = 400 + \frac{6000}{4} = 1900\text{mm} \end{cases}$$

(a) $M_u = 500\text{kN.m}$

Check the section and assume, $a = h_f$ and $\phi = 0.9$

$$M_{uf} = \phi \left[0.85f'_c b_f h_f \left(d - \frac{h_f}{2} \right) \right] = \left[0.9 \times 0.85 \times 20 \times 1200 \times 80 \left(430 - \frac{80}{2} \right) \right] \times 10^{-6}$$

$$M_{uf} = 572.83\text{kN.m} > M_u = 500\text{kN.m} \text{ (Design as rectangular section)}$$

(b) $M_u = 650\text{kN.m}$

$$M_{uf} = 572.83\text{kN.m} < M_u = 650\text{kN.m} \text{ (Design as T- section)}$$

$$A_{sf} = \frac{0.85f'_c(b_f - b_w)h_f}{f_y} = \frac{0.85 \times 20(1200 - 400) \times 80}{400} = 2720\text{mm}^2$$

$$M_{uf} = \phi A_{sf} f_y \left(d - \frac{h_f}{2} \right) = \left[0.9 \times 2720 \times 400 \left(430 - \frac{80}{2} \right) \right] \times 10^{-6} = 382\text{kN.m}$$

$$M_{uw} = M_{u\text{ext}} - M_{uf} = 650 - 382 = 268\text{kN.m}$$

$$R = \frac{M_{uw}}{\phi b_w d^2} = \frac{268 \times 10^6}{0.9 \times 400 \times 430^2} = 4.0262, \quad m = \frac{f_y}{0.85f'_c} = \frac{400}{0.85 \times 20} = 23.53$$

$$\rho = \frac{1}{m} \left(1 - \sqrt{1 - \frac{2mR}{f_y}} \right) = \frac{1}{23.53} \left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 4.0262}{400}} \right) = 0.01167$$

$$A_{sw} = \rho b_w d = 0.01167 \times 400 \times 430 = 2007\text{mm}^2$$

$$A_s = A_{sf} + A_{sw} = 2720 + 2007 = 4727 \text{ mm}^2$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{4727}{400 \times 430} = 0.0275 \quad \rho_f = \frac{A_{sf}}{b_w d} = \frac{2720}{400 \times 430} = 0.01581$$

$$\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_{uc}}{\epsilon_{uc} + 0.004} = 0.85 \times 0.85 \times \frac{20}{400} \frac{0.003}{0.003 + 0.004} = 0.015482$$

$$\rho_{w)max} = \rho_{max} + \rho_f = 0.015482 + 0.01581 = 0.03129$$

$$\rho_w = 0.0275 \leq \rho_{w)max} = 0.03129 \quad \text{ok}$$

$$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} = \frac{\sqrt{20}}{4 \times 400} = 0.002795 \geq \frac{1.4}{f_y} = \frac{1.4}{400} = 0.0035$$

$$\rho_{min} = 0.0035$$

$$\rho_{min} < \rho_w < \rho_{w)max} \quad \text{ok}$$

Check ϕ

$$a = \frac{A_{sw} f_y}{0.85 f'_c b_w} = \frac{2007 \times 400}{0.85 \times 20 \times 400} = 118.1 \text{ mm} \quad c = \frac{a}{\beta_1} = \frac{118.1}{0.85} = 138.9 \text{ mm}$$

$$\epsilon_t = \frac{d-c}{c} \epsilon_{cu} = \frac{430-138.9}{138.9} \times 0.003 = 0.0063 > 0.005 \quad \rightarrow \phi = 0.9 \quad \text{ok}$$

$$\text{Assume } \phi b = 28 \text{ mm} \quad \text{No. of bars} = \frac{A_s}{A_b} = \frac{4727}{616} = 7.67 \approx 8 \quad \text{use two layers}$$

$$S = \frac{b - 2\text{cover} - 2\phi_{stirrup} - n\phi b}{n - 1}$$

$$S = \frac{400 - 2 \times 40 - 2 \times 10 - 4 \times 16}{3} = 627 \text{ mm} \geq 25 \text{ mm}, \phi b = 28 \text{ mm}, \frac{4}{3} \text{ M.A.S.} \quad \text{ok}$$

$$(c) M_u = 800 \text{ kN.m},$$

$$M_{uf} = 572.83 \text{ kN.m} < M_u = 650 \text{ kN.m} \quad (\text{Design as T-section})$$

$$A_{sf} = \frac{0.85 f'_c (b_f - b_w) h_f}{f_y} = \frac{0.85 \times 20 (1200 - 400) \times 80}{400} = 2720 \text{ mm}^2$$

$$M_{uf} = \phi A_{sf} f_y \left(d - \frac{h_f}{2} \right) = \left[0.9 \times 2720 \times 400 \left(430 - \frac{80}{2} \right) \right] \times 10^{-6} = 382 \text{ kN.m}$$

$$M_{uw} = M_{u)ext} - M_{uf} = 800 - 382 = 418 \text{ kN.m}$$

$$R = \frac{M_{uw}}{\phi b_w d^2} = \frac{418 \times 10^6}{0.9 \times 400 \times 430^2} = 6.2797, \quad m = \frac{f_y}{0.85 f'_c} = \frac{400}{0.85 \times 20} = 23.53$$

$$\rho = \frac{1}{m} \left(1 - \sqrt{1 - \frac{2mR}{f_y}} \right) = \frac{1}{23.53} \left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 6.2797}{400}} \right) = 0.019151$$

$$A_{sw} = \rho b_w d = 0.019151 \times 400 \times 430 = 3294 \text{ mm}^2$$

$$A_s = A_{sf} + A_{sw} = 2720 + 3294 = 6014 \text{ mm}^2$$

$$\rho_w = \frac{A_s}{b_w d} = \frac{6014}{400 \times 430} = 0.03497 \quad \rho_f = \frac{A_{sf}}{b_w d} = \frac{2720}{400 \times 430} = 0.01581$$

$$\rho_{max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_{uc}}{\epsilon_{uc} + 0.004} = 0.85 \times 0.85 \times \frac{20}{400} \frac{0.003}{0.003 + 0.004} = 0.015482$$

$$\rho_{w)max} = \rho_{max} + \rho_f = 0.015482 + 0.01581 = 0.03129$$

$$\rho_w = 0.03497 > \rho_{w)max} = 0.03129 \quad \text{No governing}$$

Use one of the following options:

- d- Increase flange thickness (h_f)
- e- Use compression steel reinforcement (doubly reinforced section)
- f- Increase the effective depth (d)

Repeat the solution increase the effective depth (d)

Use $d=500\text{mm}$