Almustafa University College

Civil Engineering Department

Mathematic II

Second Stage

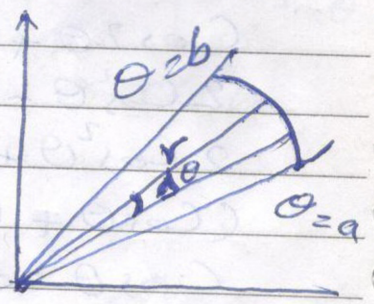
Area in Polar Coordinates

Area in Polar Coordinates

If (f) is continuous and $f(\theta) \geq 0$ on $[a, b]$ where $0 \leq a \leq b \leq 2\pi$ then the area (A) of the region bounded by the graph of $r = f(\theta)$, $\theta = a$ & $\theta = b$

$$A = \int_{\theta=a}^{\theta=b} \frac{1}{2}[f(\theta)]^2 \, d\theta = \int_a^b \frac{1}{2} r^2 \, d\theta$$

$$\left.\begin{aligned} A &= \tfrac{1}{2} r^2 \Delta\theta \\ &\sum \tfrac{1}{2} r^2 \Delta\theta \\ &\int_a^b \tfrac{1}{2} r^2 \, d\theta = \end{aligned}\right\} \Longrightarrow$$



Ex) Find the area of the region that is bounded by the cardioid $r = 2 + 2\cos\theta$

Sol

$$A = 2\int_0^{\pi} \frac{1}{2} r^2 \, d\theta$$

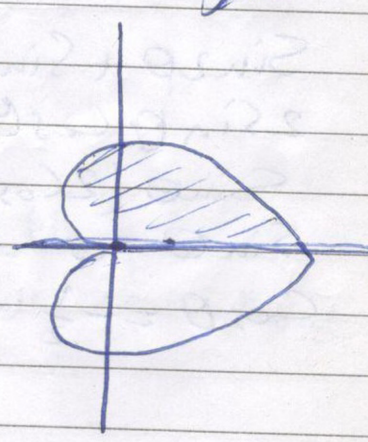
$$= \int_0^{\pi} (2 + 2\cos\theta)^2 \, d\theta$$

$$= \int_0^{\pi} (4 + 8\cos\theta + 4\cos^2\theta) \, d\theta$$

$$= \int_0^{\pi} (4\theta + 8\sin\theta +$$

$$= 6\pi$$

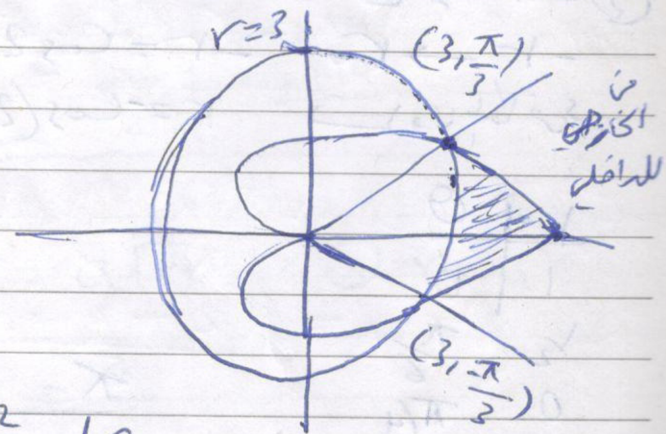
مثلاً



Let f & g be continuous functions such that $f(\theta) \geqslant g(\theta) \geqslant 0$ for every θ in $[a, b]$, where $0 \leqslant a \leqslant b \leqslant 2\pi$, let R denoted by region bounded by the graph $r = f(\theta)$, $r = g(\theta)$, $\theta = a$ & $\theta = b$ the area A & y is

$$A = \frac{1}{2}\int_a^b [f(\theta)]^2 - [g(\theta)]^2 \, d\theta$$

Ex) Find the area of the region that inside the cardioid $r = 2 + 2\cos\theta$ and out side the circle $r = 3$



$$3 = 2 + 2\cos\theta$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{3}$$
$$r = 3$$

$$A = \frac{1}{2}\int_{-\pi/3}^{\pi/3} (2 + 2\cos\theta)^2 - (3)^2 \, d\theta$$

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3} 4 + 8\cos\theta + 4\cos^2\theta - 9 \, d\theta$$

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3} 4\underbrace{\cos^2\theta}_{\frac{1}{2}(1+\cos 2\theta)} + 8\cos\theta - 5 \, d\theta$$

$$= \frac{1}{2}\left[\frac{4}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + 8\sin\theta - 5\theta\right]_{-\pi/3}^{\pi/3} = \frac{9\sqrt{3}}{2} - \pi$$

Ex) Find the area of the region enclosed by the rose $r = \cos 2\theta$

Sol

$r = 0 \Rightarrow 2\theta = \frac{\pi}{2} \rightarrow \theta = \frac{\pi}{4}$

$r = 1 \Rightarrow 2\theta = 0 \Rightarrow \theta = 0$

Sym.

① $\theta \rightarrow -\theta \qquad r = \cos 2(-\theta)$

$\therefore$ Sym. about x

② $-r \rightarrow r \qquad -r = \cos 2\theta$

تساوي المعادلة $\quad r = -\cos 2\theta \quad$ Sym. about origin

③ $-\theta \rightarrow \theta$

$-r \rightarrow r \qquad -r = \cos 2(-\theta)$

تساوي المعادلة $\quad r = -\cos(2\theta) \quad$ Sym. about y

eq. 1 $\cancel{r = \cos 2(\theta \ldots)}$

$r = \cos 2(\theta + 2n\pi)$

$= \cos 2\theta$

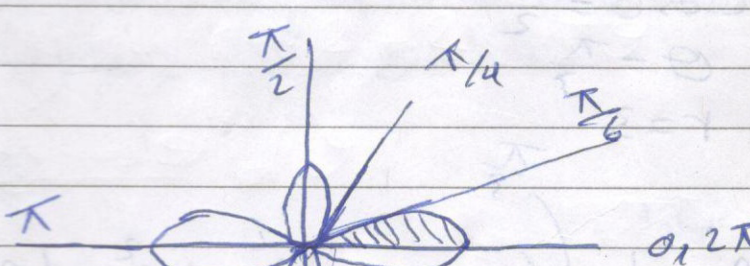
eq. 2

$-r = \cos 2(\theta + \pi + 2n\pi)$

$r = \cos(2\theta + 2\pi + 2n\pi)$

$r = -\cos 2\theta$

r	θ
1	0
$1/2$	$\pi/6$
0	$\pi/4$
-1	$\pi/2$
1	π
-1	$3\pi/2$



ناخذ جزء من الى $\frac{\pi}{4}$ ونضرب ب 8 لاستنتج المساحة الكلية

$$A = 8\int_0^{\pi/4} \frac{1}{2}(\cos 2\theta)^2\, d\theta = 4\int_0^{\pi/4} \frac{(1 + \cos 4\theta)}{2}\, d\theta$$

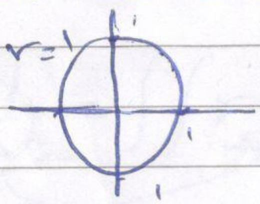
$$= 4\left[\theta + \frac{1}{4}(\sin 4\theta)\right]_0^{\pi/4} = \pi$$

H.W

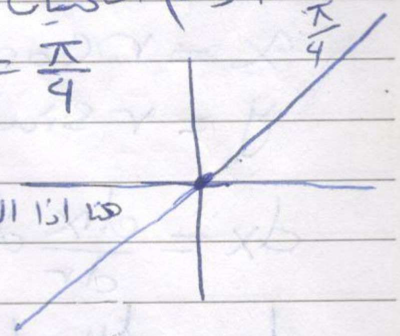
Find the region between $r = 1 - \cos\theta$ & $r = 1 + \cos\theta$

Ex) Sketch the curves ا = اسم المنحنيات

(a) $r = 1$



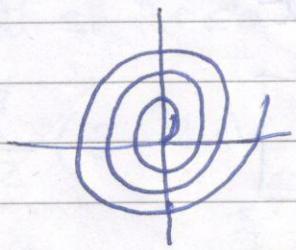
(b) $\theta = \frac{\pi}{4}$



هنا اذا الـ r سالب أو موجب

(c) $r = \theta$

θ	r
0	0
$\frac{\pi}{4}$	$\frac{3.14}{4}$
$\frac{\pi}{2}$	$\frac{3.14}{2}$
π	3.14



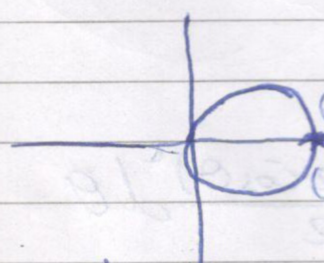
θ	r
$\frac{3\pi}{2}$	$\frac{3 \times 3.14}{2}$
2π	2×3.14
$\frac{5\pi}{2}$	$\frac{5 \times 3.14}{2}$

H.W: sketch

(1) $r = a\theta$, (2) $r = a\sqrt{\theta}$, (3) $r = ae^{b\theta}$

(4) $r = \frac{a}{\sqrt{\theta}}$, (5) $r = \frac{a}{\theta}$

(d) $r = 2a\cos\theta$

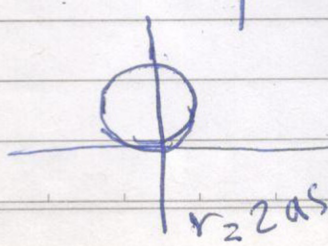


$(-2a, \pi)$

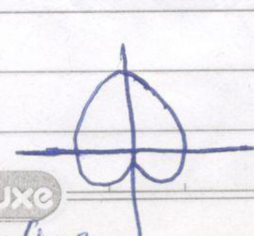
$(2a, 0)$

$r = a(1+\cos\theta)$

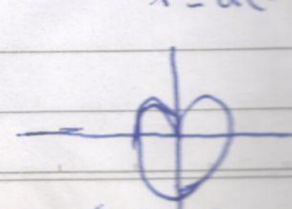
$r = a(1-\cos\theta)$



$r = 2a\sin\theta$



$r = a(1+\sin\theta)$



$r = a(1-\sin\theta)$