كلية المصطفى الجامعة

Fundamental of Electrical Engineering

Assit.Lec. Shaimaa Shukri

First lecture

1. SYSTEMS OF UNITS

In the past, the systems of units most commonly used were the English and metric, as outlined in Table 1.1. Note that while the English system is based on a single standard, the metric is subdivided into two interrelated standards: the MKS and the CGS. Fundamental quantities of these systems are compared in Table 1.1 along with their abbreviations. The MKS and CGS systems draw their names from the units of measurement used with each system; the MKS system uses Meters, Kilograms,

- and Seconds. Understandably, the use of more than one system of units in a world that finds itself continually shrinking in size, due to advanced technical developments in communications and transportation, would introduce unnecessary complications to the basic understanding of any technical data

TABLE 1.1

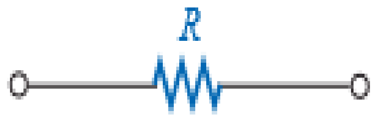
Comparison of the English and metric systems of units.

English	Metric		
	MKS	CGS	SI
Length:			
Yard (yd) (0.914 m)	Meter (m) (39.37 in.) (100 cm)	Centimeter (cm) (2.54 cm = 1 in.)	**Meter (m)**
Mass:			
Slug (14.6 kg)	Kilogram (kg) (1000 g)	Gram (g)	**Kilogram (kg)**
Force:			
Pound (lb) (4.45 N)	Newton (N) (100,000 dynes)	Dyne	**Newton (N)**
Temperature:			
Fahrenheit (°F) $\left(= \frac{9}{5}°\text{C} + 32\right)$	Celsius or Centigrade (°C) $\left(= \frac{5}{9}(°\text{F} - 32)\right)$	Centigrade (°C)	**Kelvin (K)** K = 273.15 + °C
Energy:			
Foot-pound (ft-lb) (1.356 joules)	Newton-meter (N•m) or joule (J) (0.7376 ft-lb)	Dyne-centimeter or erg (1 joule = 10^7 ergs)	**Joule (J)**
Time:			
Second (s)	Second (s)	Second (s)	**Second (s)**

Unit Sybmol	Unit	Quantity Sybmol	Baisc Quantity
m	meter	L	Length
Kg	Kilogram	M	Mass
S	Second	T	Time
A	Amper	I	Current
K	Kelvin	T	Temperature
cd	Candel	Lun	lutensity

2. Electric current and Ohms Law

- (Resistance)
- The flow of charge through any material encounters an opposing force similar in many respects to mechanical friction. This opposition, due to the collisions between electrons and between electrons and other atoms in the material, which converts electrical energy into another form of energy such as heat, is called the resistance of the material. The unit of measurement of resistance is the ohm, for which the symbol is `Ω, the capital Greek letter omega. The circuit symbol for resistance appears is shown in figure with the graphic abbreviation for resistance (R).



The resistance of any material with a uniform cross-sectional area is determined by the following four factors:

1. Material , it depends on the nature of the material .
2. Length , it varies directly as its length (L) .
3. Cross-sectional area , It varies inversely as the cross sectional area (A) of the conductor
4. Temperature, It also depends on the temperature of conductor .

- Neglecting the last factor for the time being , we can say that:

$$R \propto \frac{L}{A}$$

$$R = \rho \frac{L}{A}$$

Where :

R is the resistance of the conductor (Ω) .
L is the length of the conductor (m) .
A is the cross sectional area of the conductor (m^2) .
ρ is a constant depending on the nature of the material of the conductor and known as its specific resistance (Ω .m) .

- **Example : Calculate the resistance of 1km cable composed of 19 strands of similar alloy conductors, each strand being 1.32 mm in diameter . Resistivity of alloy may be taken as 1.72 x 10 – 8 Ω . m .**

SOL.

A = $\pi d^2 /4$

A = 3.14 x (1.32 x 10-3)2 / 4

Total cross sectional area of the cable = 19 x 13.67 x $10^{-7}\ m^2$

$$R = \rho \frac{L}{A}$$

$$= \frac{1.72 \times 10^{-8} \times 1000}{19 \times 13.67 \times 10^{-7}} = 0.66\ \Omega$$

➢ Effect of temperature on the resistanc

The resistance of a conductor depends on the temperature as follows:

$R \; \alpha \; T$

Where

R is the value of resistance .

T is the temperature of the conductor .

$R_t = R_o (1 + \alpha_o t)$

$R_2 = R_1 [1 + \alpha (t_2 - t_1)]$

Where

R_t is the resistance of the conductor at t° C .

R_o is the resistance of the conductor at 0° C .

α_o is the temperature coefficient of the conductor at 0° C

R_1 is the resistance of the conductor at t1° C .

R_2 is the resistance of the conductor at t2° C .

- Example : A lamp of 100 watt power , 240 volt reaches 2000° C . If the temperature coefficient of the lamp at 15°C is 5 x 10-3 . Calculate the resistance of the lamp at 15° C ?

Sol.

$$P = \frac{V^2}{R}, \qquad R = \frac{(240)^2}{100} = 576\ \Omega$$

$$R_2 = R_1 \{ 1 + \alpha (t_2 - t_1) \}$$

$$576 = R_1 \{ 1 + 5 \times 10^{-3} (2000 - 15) \}$$

$$R_1 = 52.7\ \Omega$$

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Fundamental of Electrical Engineering

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Second lecture

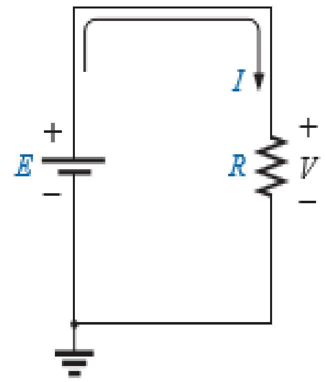
1. Ohm's Law, Power, and Energy

$$\text{Current} = \frac{\text{potential difference}}{\text{resistance}}$$

$$I = \frac{E}{R} \quad \text{(amperes, A)}$$

$$E = IR \quad \text{(volts, V)}$$

$$R = \frac{E}{I} \quad \text{(ohms, } \Omega\text{)}$$



EXAMPLE Determine the current resulting from the application of a 9-V battery across a network with a resistance of 2.2 Ω.

Solution:

$$I = \frac{E}{R} = \frac{9\text{ V}}{2.2\ \Omega} = \mathbf{4.09\ A}$$

EXAMPLE Calculate the resistance of a 60-W bulb if a current of 500 mA results from an applied voltage of 120 V.

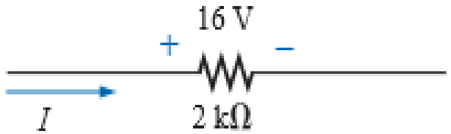
Solution:

$$R = \frac{E}{I} = \frac{120\ \text{V}}{500 \times 10^{-3}\ \text{A}} = \mathbf{240\ \Omega}$$

For an isolated resistive element, the polarity of the voltage drop is as shown in Fig. (a) for the indicated current direction. A reversal in current will reverse the polarity, as shown in Fig. (b). In general, the flow of charge is from a high (+) to a low (−) potential. Polarities as established by current direction will become increasingly important in the analysis to follow.

EXAMPLE Calculate the current through the 2-kΩ resistor of Fig. if the voltage drop across it is 16 V.

Solution:



$$I = \frac{V}{R} = \frac{16\ \text{V}}{2 \times 10^{3}\ \Omega} = \mathbf{8\ mA}$$

EXAMPLE Calculate the voltage that must be applied across the soldering iron of Fig. to establish a current of 1.5 A through the iron if its internal resistance is 80 Ω.

Solution:

$$E = IR = (1.5\ \text{A})(80\ \Omega) = \mathbf{120\ V}$$

2.power

Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, that is, a rate of doing work. For instance, a large motor has more power than a small motor because it can convert more electrical energy into mechanical energy in the same period of time. Since converted energy is measured in joules (J) and time in seconds (s), power is measured in joules/second (J/s). The electrical unit of measurement for power is the watt (W), defined by

$$1 \text{ watt (W)} = 1 \text{ joule/second (J/s)}$$

In equation form, power is determined by

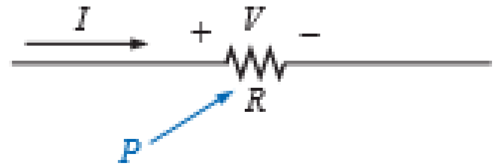
$$P = \frac{W}{t} \quad \text{(watts, W, or joules/second, J/s)}$$

$$1 \text{ horsepower} \cong 746 \text{ watts}$$

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$ و $$I = \frac{Q}{t}$$ ⟹ $$\boxed{P = VI}$$ (watts)

$$P = VI = V\left(\frac{V}{R}\right)$$

and $$\boxed{P = \frac{V^2}{R}}$$ (watts)



or $$P = VI = (IR)I$$

and $$\boxed{P = I^2R}$$ (watts)

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$

- The magnitude of the power delivered or absorbed by a battery is given by:

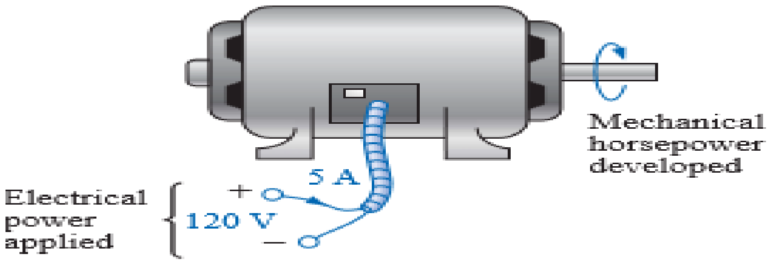
$$\boxed{P = EI}$$ (watts)

with E the battery terminal voltage and I the current through the source.

EXAMPLE Find the power delivered to the dc motor of Fig.

Solution:

$$P = VI = (120\text{ V})(5\text{ A}) = 600\text{ W} = \mathbf{0.6\ kW}$$



EXAMPLE What is the power dissipated by a 5-Ω resistor if the current is 4 A?

Solution:

$$P = I^2R = (4\text{ A})^2(5\ \Omega) = \mathbf{80\ W}$$

EXAMPLE: Determine the current through a 5-k̀Ω resistor when the power dissipated by the element is 20 mW.**?**

Solution:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{20 \times 10^{-3}\text{ W}}{5 \times 10^{3}\ \Omega}} = \sqrt{4 \times 10^{-6}} = 2 \times 10^{-3}\text{ A}$$
$$= \mathbf{2\ mA}$$

3. Series Circuits

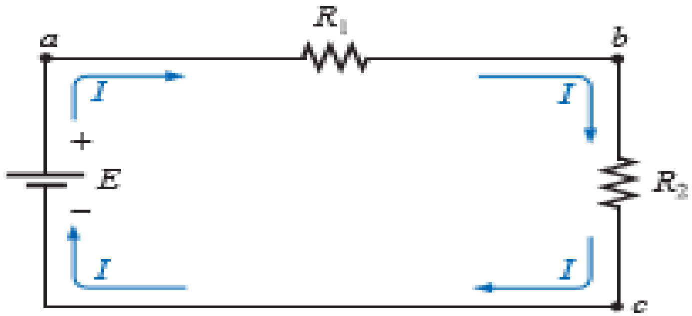
A circuit consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 1 has three elements joined at three terminal points (a, b, and c) to provide a closed path for the current I.

Two elements are in series if

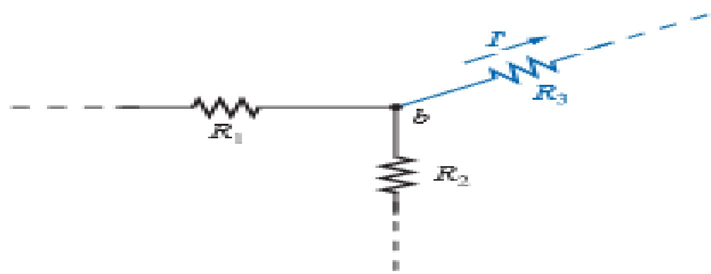
1. They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).

2. The common point between the two elements is not connected to another current-carrying element.

❖ **The current is the same through series elements.**



(a) Series circuit



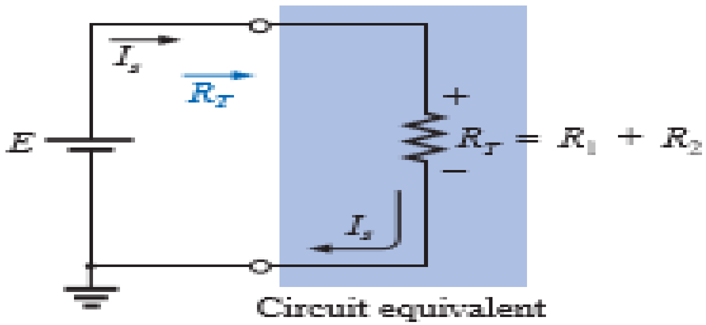
(b) R_1 and R_2 are not in series

(a) Series circuit; (b) situation in which R1 and R2 are not in series.

The total resistance of a series circuit is the sum of the resistance levels

$$R_T = R_1 + R_2 + R_3 + \cdots + R_N \quad \text{(ohms, } \Omega\text{)}$$

$$I_s = \frac{E}{R_T} \quad \text{(amperes, A)}$$



The voltage across each resistor using Ohms law; that is :

$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3,\ \ldots,\ V_N = IR_N \quad \text{(volts, V)}$$

The power delivered to each resistor can then be determined using any one of three equations as listed below for R1:

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \quad \text{(watts, W)}$$

The power delivered by the source is

$$P_{\text{del}} = EI \quad \text{(watts, W)}$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

That is,

$$P_{del} = P_1 + P_2 + P_3 + \cdots + P_N$$

EXAMPLE:

a. Find the total resistance for the series circuit of Fig.
b. Calculate the source current Is.
c. Determine the voltages V1, V2, and V3.
d. Calculate the power dissipated by R1, R2, and R3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d)

Solutions:

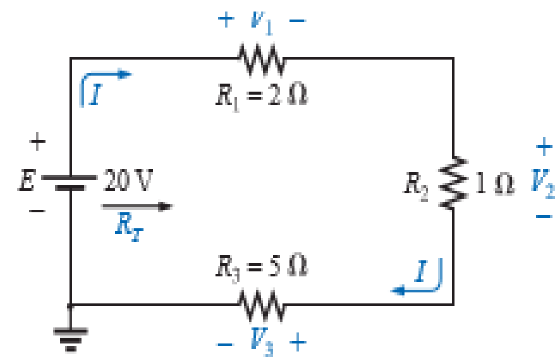
a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ V}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\ A)(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\ A)(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\ A)(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1 I_1 = (5\ V)(2.5\ A) = \mathbf{12.5\ W}$
$P_2 = I_2^2 R_2 = (2.5\ A)^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\ V)^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{del} = EI = (20\ V)(2.5\ A) = \mathbf{50\ W}$
$P_{del} = P_1 + P_2 + P_3$
$50\ W = 12.5\ W + 6.25\ W + 31.25\ W$
$50\ W = 50\ W$ (checks)



EXAMPLE :

Determine RT, I, and V2 for the circuit of as shown in Figure:

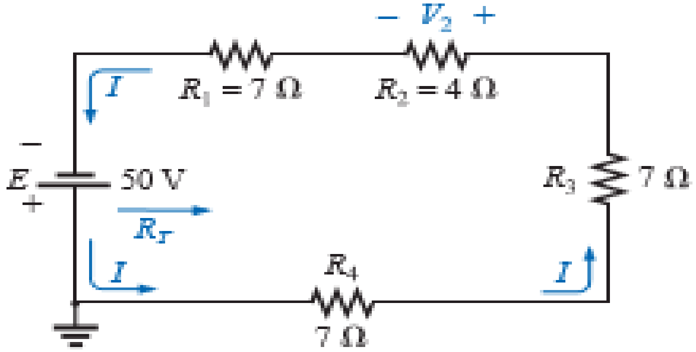
Solution:

Note the current direction as established by the battery and the polarity of the voltage drops across R2 as determined by the current direction. Since R1 = R3 = R4,

$$R_T = NR_1 + R_2 = (3)(7\ \Omega) + 4\ \Omega = 21\ \Omega + 4\ \Omega = \mathbf{25\ \Omega}$$

$$I = \frac{E}{R_T} = \frac{50\ \text{V}}{25\ \Omega} = \mathbf{2\ A}$$

$$V_2 = IR_2 = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$



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Third lecture

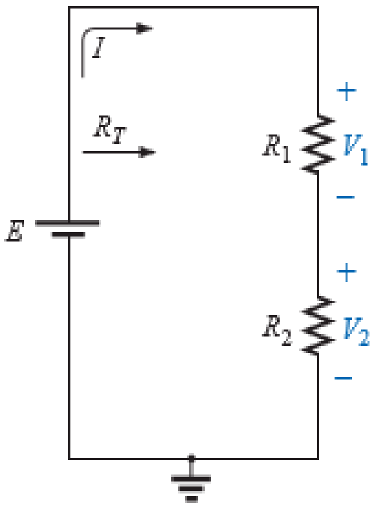
VOLTAGE DIVIDER RULE

In a series circuit,

the voltage across the resistive elements will divide as the magnitude of the resistance levels. The rule can be derived by analyzing the network of Figure:

$$R_T = R_1 + R_2$$

- and $$I = \frac{E}{R_T}$$



Applying Ohm's law:

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right)R_1 = \frac{R_1E}{R_T}$$

with $$V_2 = IR_2 = \left(\frac{E}{R_T}\right)R_2 = \frac{R_2E}{R_T}$$

Note that the format for V_1 and V_2 is

$$V_x = \frac{R_xE}{R_T}$$ (voltage divider rule)

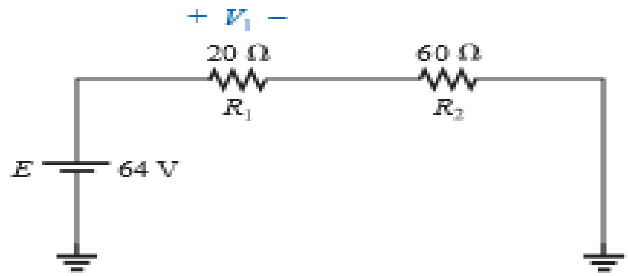
In words, the **voltage divider** rule states that the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

EXAMPLE 1 :

Determine the voltage V1 for the network of Figure

solution:

$$V_1 = \frac{R_1E}{R_T} = \frac{R_1E}{R_1 + R_2} = \frac{(20\ \Omega)(64\ \text{V})}{20\ \Omega + 60\ \Omega} = \frac{1280\ \text{V}}{80} = \mathbf{16\ V}$$



EXAMPLE 2:

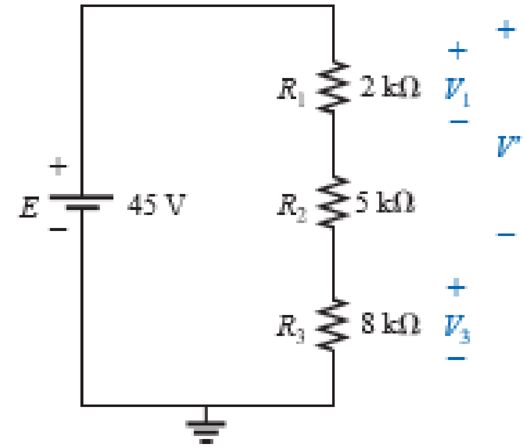
Using the voltage divider rule, determine the voltages V1 and V3 for the series circuit of Figure

$$V_1 = \frac{R_1E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \mathbf{6\ V}$$

$$V_3 = \frac{R_3E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

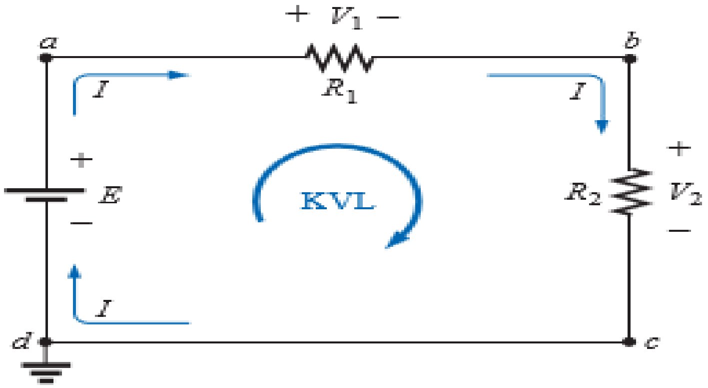
$$= \frac{360\ \text{V}}{15} = \mathbf{24\ V}$$



KIRCHHOFF'S VOLTAGE LAW

- Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

$$\Sigma_C V = 0$$



$$+E - V_1 - V_2 = 0$$

or

$$E = V_1 + V_2$$

the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

Kirchhoff's voltage law can also be stated in the following form:

$$\Sigma_C V_{\text{rises}} = \Sigma_C V_{\text{drops}}$$

EXAMPLE 1:

For the circuit of Figure:

a. Find RT.

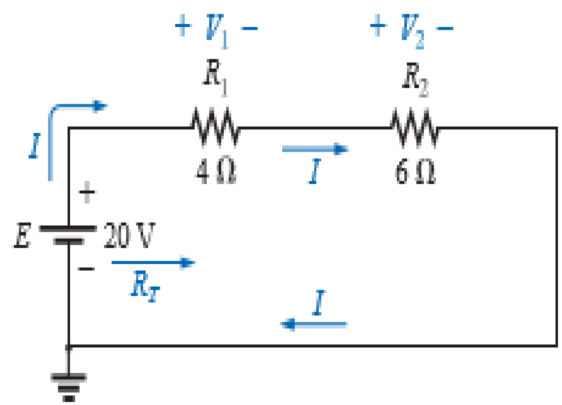
b. Find I.

c. Find V1 and V2.

d. Find the power to the 4-`Ω and 6- `Ω resistors.

e. Find the power delivered by the battery, and compare it to that dissipated by the 4- `Ω and 6- `Ω resistors combined.

f. Verify Kirchhoff’s voltage law (clockwise direction).



a. $R_T = R_1 + R_2 = 4\ \Omega + 6\ \Omega = \mathbf{10\ \Omega}$

b. $I = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{10\ \Omega} = \mathbf{2\text{ A}}$

c. $V_1 = IR_1 = (2\text{ A})(4\ \Omega) = \mathbf{8\text{ V}}$
$V_2 = IR_2 = (2\text{ A})(6\ \Omega) = \mathbf{12\text{ V}}$

d. $P_{4\Omega} = \dfrac{V_1^2}{R_1} = \dfrac{(8\text{ V})^2}{4} = \dfrac{64}{4} = \mathbf{16\text{ W}}$

$P_{6\Omega} = I^2R_2 = (2\text{ A})^2(6\ \Omega) = (4)(6) = \mathbf{24\text{ W}}$

e. $P_E = EI = (20\text{ V})(2\text{ A}) = \mathbf{40\text{ W}}$
$P_E = P_{4\Omega} + P_{6\Omega}$
$40\text{ W} = 16\text{ W} + 24\text{ W}$
$40\text{ W} = 40\text{ W}$ (checks)

f. $\Sigma_{\circlearrowright} V = +E - V_1 - V_2 = 0$
$E = V_1 + V_2$
$20\text{ V} = 8\text{ V} + 12\text{ V}$
$20\text{ V} = 20\text{ V}$ (checks)

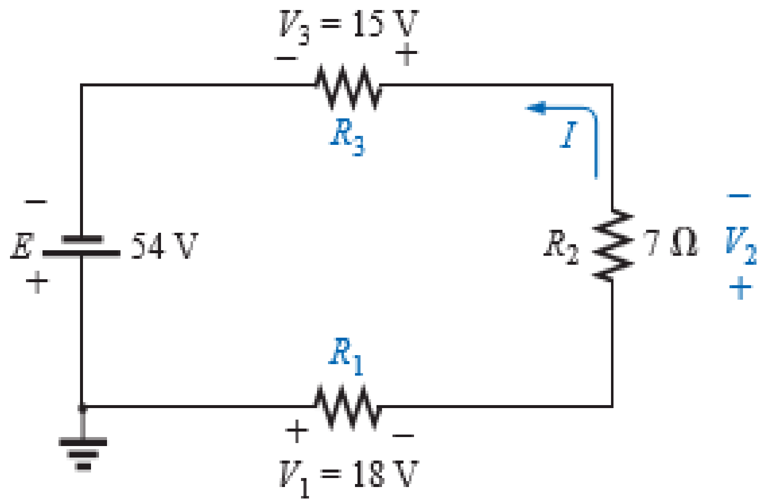
EXAMPLE 2:

For the circuit of Figure

a. Determine V2 using Kirchhoff's voltage law.

b. Determine I.

c. Find R1 and R3.



Solutions*: a*. Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or

$$E = V_1 + V_2 + V_3$$

and $V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\text{ V}}$

b. $I = \frac{V_2}{R_2} = \frac{21\text{ V}}{7\ \Omega} = \mathbf{3\text{ A}}$

c. $R_1 = \frac{V_1}{I} = \frac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

$R_3 = \frac{V_3}{I} = \frac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

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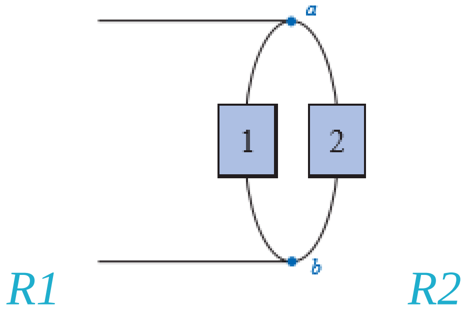
Forth lecture

PARALLEL CIRCUIT

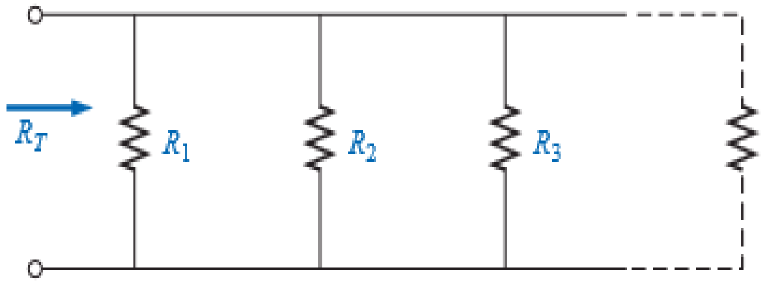
- Two elements, branches, or networks are in parallel if they have two points in common

$$\frac{1}{RT} = \frac{1}{R1} + \frac{1}{R2}$$



- The total resistance for the network can be determined by direct substitution into Eq.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$



CONDUCTANCE

For parallel elements, the total conductance is the sum of the individual conductance

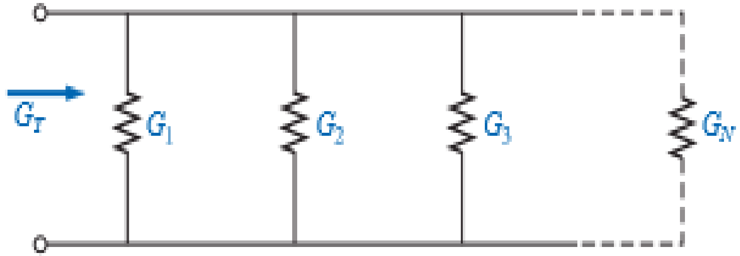
The conductance [G] is definde as:

$$G = \frac{1}{R} \quad \text{simenes}(S)$$

So ,we can write the total conductance GT for the parallel cct shown ,as :

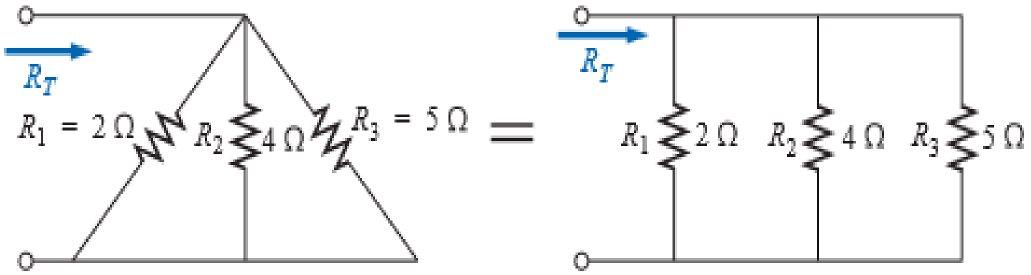
GT= G1+ G2+ G3+GN

$$RT = \frac{1}{GT}$$



EXAMPLE 1:

Determine the total resistance for the network of Figure:



Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2\ \Omega} + \frac{1}{4\ \Omega} + \frac{1}{5\ \Omega} = 0.5\ \text{S} + 0.25\ \text{S} + 0.2\ \text{S}$$

$$= 0.95\ \text{S}$$

and $$R_T = \frac{1}{0.95\ \text{S}} = \mathbf{1.053\ \Omega}$$

EXAMPLE 2 :

a. Find the total resistance of the network of Fig. 1

b. Calculate the total resistance for the network of Fig. 2

Solution:

a. For R1=R2=R3 THEN

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = 4\ \Omega$$

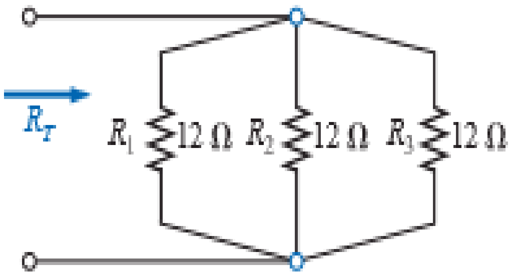


figure 1

b.

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = 0.5\ \Omega$$

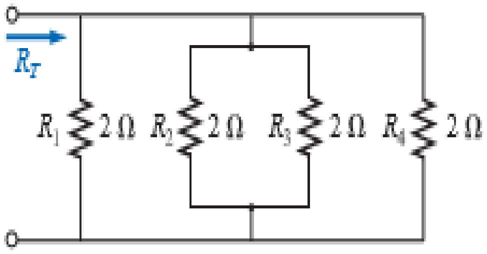
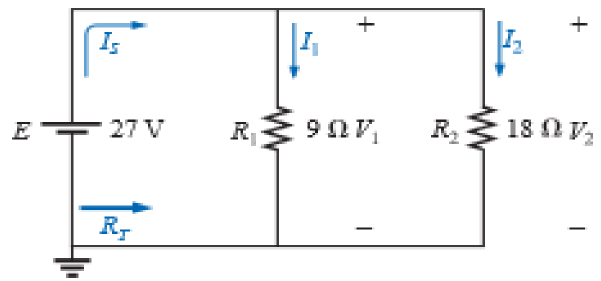


figure 2

- **PARALLEL CIRCUITS:**

EXAMPLE 3:



For the parallel network of Fig.

a. Calculate RT.

b. Determine Is.

c. Calculate I1 and I2, and demonstrate that Is =I1 + I2.

d. Determine the power to each resistive load.

e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

Solutions

a. $R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \frac{162\ \Omega}{27} = \mathbf{6\ \Omega}$

b. $I_s = \frac{E}{R_T} = \frac{27\ \text{V}}{6\ \Omega} = \mathbf{4.5\ A}$

c. $I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27\ \text{V}}{9\ \Omega} = \mathbf{3\ A}$

$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27\ \text{V}}{18\ \Omega} = \mathbf{1.5\ A}$

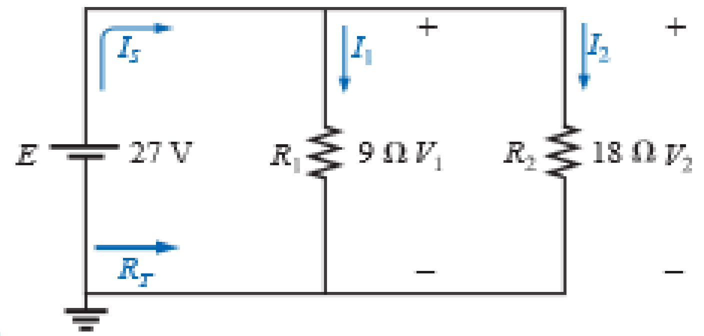
$I_s = I_1 + I_2$

$4.5\ \text{A} = 3\ \text{A} + 1.5\ \text{A}$

$\mathbf{4.5\ A = 4.5\ A}$ (checks)

d. $P_1 = V_1 I_1 = E I_1 = (27\ \text{V})(3\ \text{A}) = \mathbf{81\ W}$
$P_2 = V_2 I_2 = E I_2 = (27\ \text{V})(1.5\ \text{A}) = \mathbf{40.5\ W}$

e. $P_s = E I_s = (27\ \text{V})(4.5\ \text{A}) = \mathbf{121.5\ W}$
$= P_1 + P_2 = 81\ \text{W} + 40.5\ \text{W} = \mathbf{121.5\ W}$



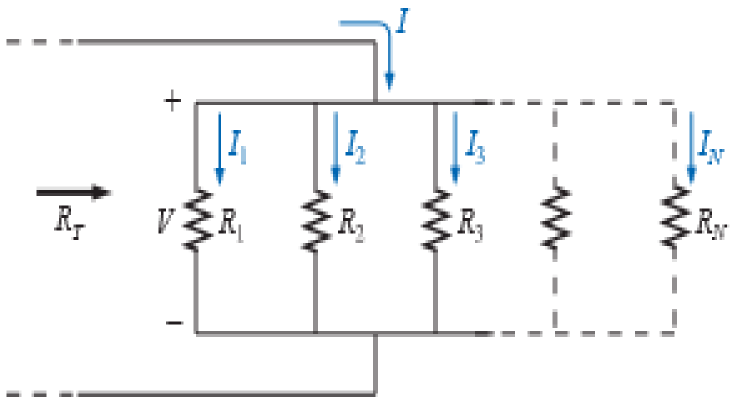
CURRENT DIVIDER RULE

- As the name suggests, the current divider rule (CDR) will determine how the current entering a set of parallel branches will split between the elements.
- For two parallel elements of equal value, the current will divide equally.
- For parallel elements with different values, the smaller the resistance, the greater the share of input current.
- For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

$$I = \frac{V}{R_T} = \frac{I_x R_x}{R_T}$$

And

$$I_x = \frac{R_T}{R_x} I$$



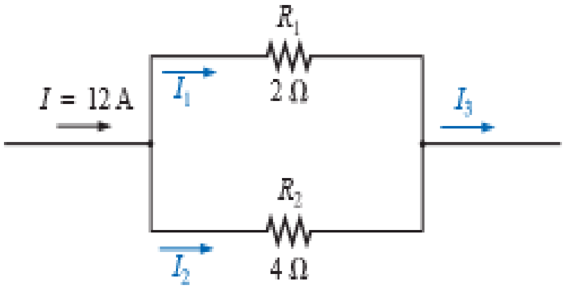
$$I_1 = \frac{R_T}{R_1} I$$

and for I_2,

$$I_2 = \frac{R_T}{R_2} I$$

and so on.

EXAMPLE1: Determine the magnitude of the currents I1, I2, and I3 for the network of Figure .



***Solution**:* the current divider rule

$$I_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\ \Omega)(12\ \text{A})}{2\ \Omega + 4\ \Omega} = \mathbf{8\ A}$$

Applying Kirchhoff's current law,

$$I = I_1 + I_2$$

and

$$I_2 = I - I_1 = 12\ \text{A} - 8\ \text{A} = \mathbf{4\ A}$$

or, using the current divider rule again,

$$I_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(2\ \Omega)(12\ \text{A})}{2\ \Omega + 4\ \Omega} = \mathbf{4\ A}$$

The total current entering the parallel branches must equal that leaving. Therefore,

$$I_3 = I = \mathbf{12\ A}$$

or

$$I_3 = I_1 + I_2 = 8\ \text{A} + 4\ \text{A} = \mathbf{12\ A}$$

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Fundamental of Electrical Engineering

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KIRCHHOFF’S CURRENT LAW

Kirchhoff’s current law (KCL) states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero.
In other words,

the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction.
In equation form:

$$\Sigma\ I_{\text{entering}} = \Sigma\ I_{\text{leaving}}$$

EXAMPLE1:

Determine the currents I3 and I4 of Fig. using Kirchhoff's current law.

Solution:

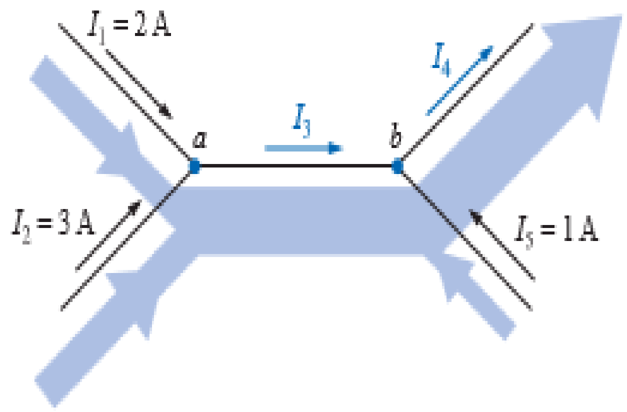
We must first work with junction a since the only unknown is I3. At junction b there are two unknowns, and both cannot be determined from one application of the law

At a:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3$$
$$I_3 = \mathbf{5\ A}$$

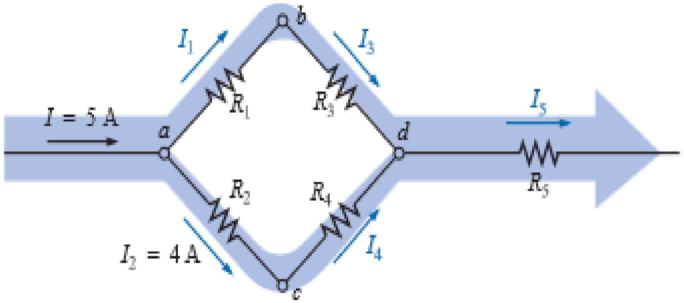
At b:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4$$
$$I_4 = \mathbf{6\ A}$$



EXAMPLE2 :

Determine I1, I3, I4, and I5 for the network of Figuer.



Solution: At a:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$

Subtracting 4 A from both sides gives

$$5\text{ A} - 4\text{ A} = I_1 + 4\not{A} - 4\not{A}$$
$$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\text{ A}}$$

At b:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I_1 = I_3 = \mathbf{1\text{ A}}$$

as it should, since R_1 and R_3 are in series and the current is the same in series elements.

At c:

$$I_2 = I_4 = \mathbf{4\text{ A}}$$

for the same reasons given for junction b.

At d:

$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5$$
$$I_5 = \mathbf{5\text{ A}}$$

If we enclose the entire network, we find that the current entering is $I = 5$ A; the net current leaving from the far right is $I_5 = 5$ A. The two must be equal since the net current entering any system must equal that leaving.

Open and short circuit in series circuits

1. Open circuit : In this case there is no current flows through the circuit as shown in fig. 1 .

   then   I = 0

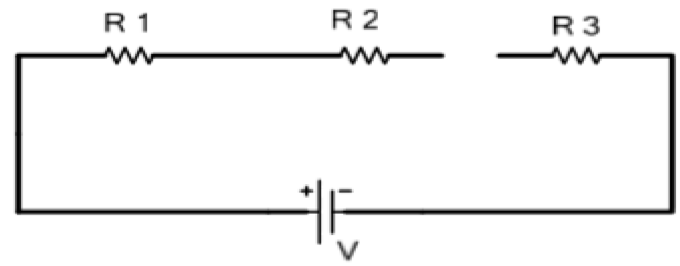


Fig. 1

2. Short circuit :

If the resistance is short circuited , the current will flow through the short circuit (no current flows through the shorted resistance) as shown in fig. 2 .

$$I = \frac{V}{R1 + R2}$$

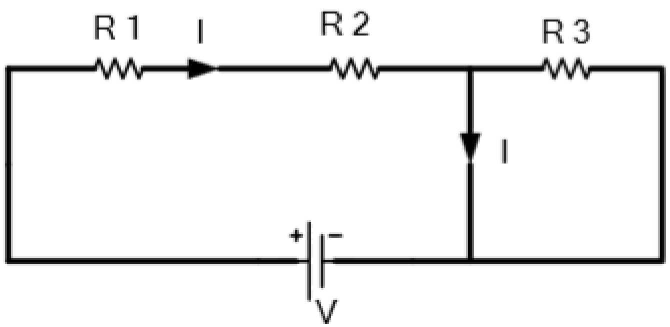


Fig.2

Open and short circuit in parallel circuits :

1. Open circuit :

In this case , there is no current flow in the open branc

$I_2 = 0$

$I = I_1 + I_3$

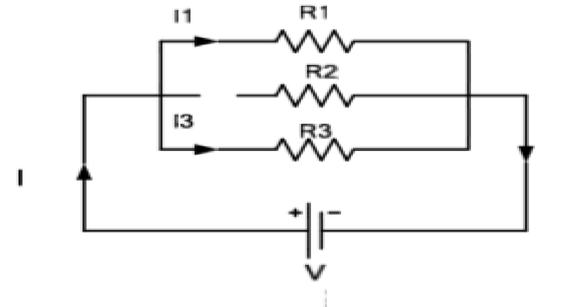


Fig. 3

2. short circuit :

In this case , there is no current flow through R1 , R2 and R3 because the total current (I) pass through the short circuit as shown in fig. 4

$$I = \frac{V}{ri}$$

Where ri is the internal resistance of the battery .

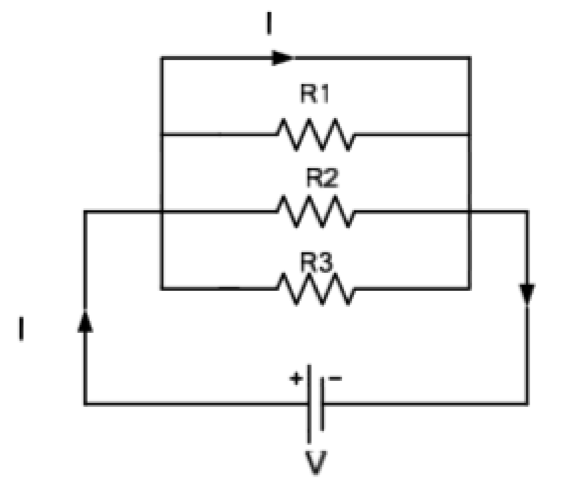


Fig. 4

Nodal method

In this method, every junction in the network where three or more branches meet is regarded as a node. One of these is regarded as the reference node (or zero potential node). Consider the circuit in fig. 1 which has three nodes . Node 3 has been taken as the reference node . VA represent the potential of node 1 with respect to node 3 . VB represent the potential of node 2 with respect to node 3 .

Node 1 :

$$V_A \left\{ \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right\} - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0$$

Node 2 :

$$V_B \left\{ \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right\} - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0$$

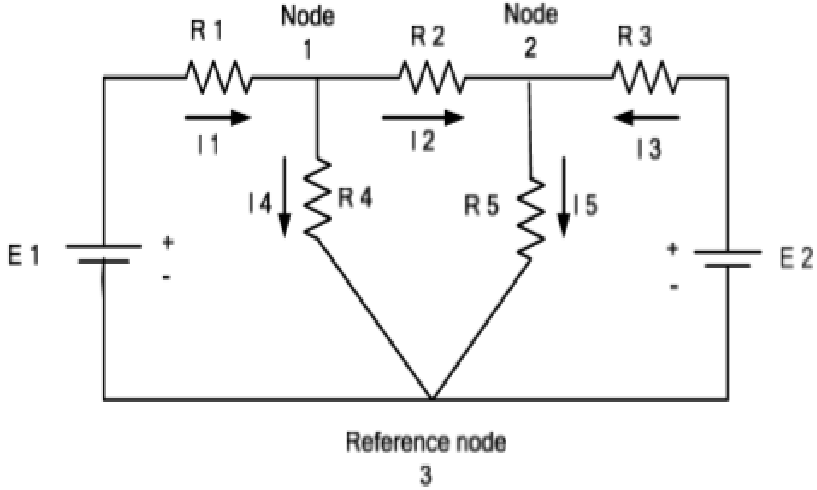


Fig. 1

Example :

Using nodal method , find all currents for the circuit shown in fig. 2 .

Consider node 3 as reference node .
Node 1 :

Node 1 :

$$V_1 \left\{ \frac{1}{1} + \frac{1}{1} + \frac{1}{0.5} \right\} - \frac{V_2}{0.5} - \frac{15}{1} = 0$$

$$4 V_1 - 2 V_2 = 15 \quad \text{-------} \quad (1)$$

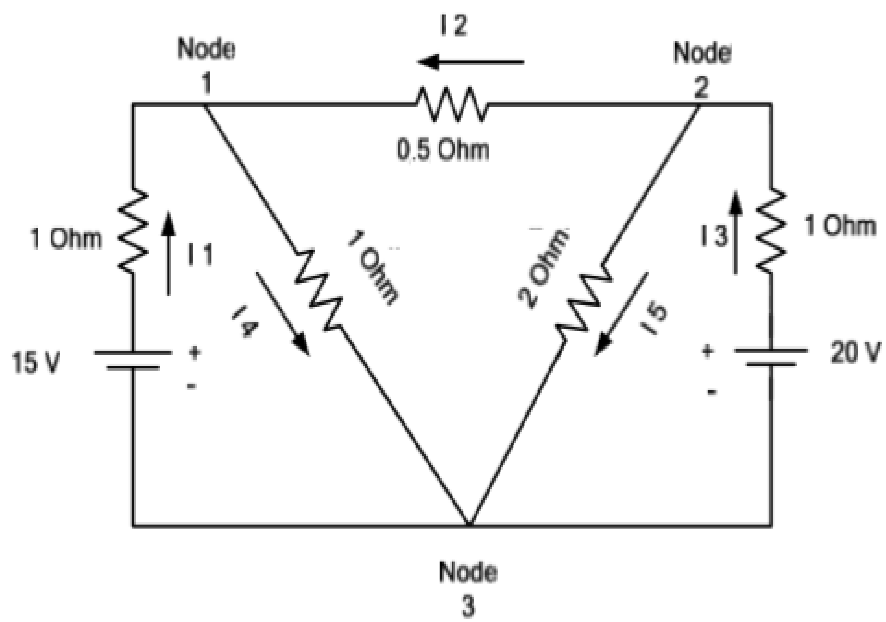


Fig. 2

Node 2 :

$$V_2 \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{0.5} \right\} - \frac{V_1}{0.5} - \frac{20}{1} = 0$$

$3.5\, V_2 - 2V_1 = 20$ -------- (2)

From Equations (1) and (2)

$V_1 = 9.25$ v , $V_2 = 11$ v

$$I_1 = \frac{15 - 9.25}{1} = 5.75$$

$$I_2 = \frac{11 - 9.25}{0.5} = 3.5 \text{ A}$$

$$I_3 = \frac{20 - 11}{1} = 9 \text{ A}$$

$I_4 = 5.75 + 3.5 = 9.25$ A

$I_5 = 9 - 3.5 = 5.5$ A

EXAMPLE 1:

Apply the branch-current method to the network of Figure to find all current in the circuit?

Solution 1:

Step 1: Since there are three distinct branches (cda, cba, ca), three currents of arbitrary directions (I1, I2, I3) are chosen, as indicated in Figure. The current directions for I1 and I2 were chosen to match the "pressure" applied by sources E1 and E2, respectively. Since both I1 and I2 enter node a, I3 is leaving.

Step 2: Polarities for each resistor are drawn to agree with assumed current directions, as indicated in Figure.

Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

Rise in potential: $+E_1$

$$\text{loop 1:}\quad \Sigma_{C} V = +E_1 - V_{R_1} - V_{R_3} = 0$$

Drop in potential: V_{R_1}, V_{R_3}

Rise in potential: $+V_{R_3}$, $+V_{R_2}$

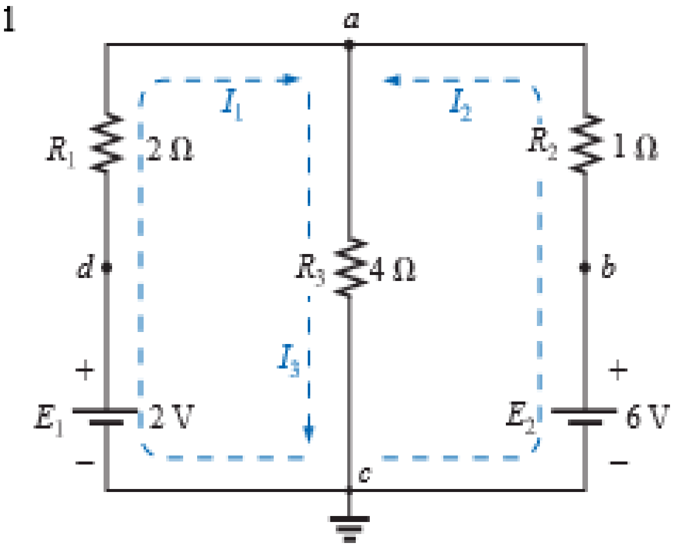
$$\text{loop 2:}\quad \Sigma_{C} V = +V_{R_3} + V_{R_2} - E_2 = 0$$

Drop in potential: E_2

and

$$\text{loop 1:}\quad \Sigma_{C} V = \underbrace{+2\text{ V}}_{\text{Battery potential}} \underbrace{-(2\ \Omega)I_1}_{\text{Voltage drop across 2-}\Omega\text{ resistor}} \underbrace{-(4\ \Omega)I_3}_{\text{Voltage drop across 4-}\Omega\text{ resistor}} = 0$$

$$\text{loop 2:}\quad \Sigma_{C} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$



Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

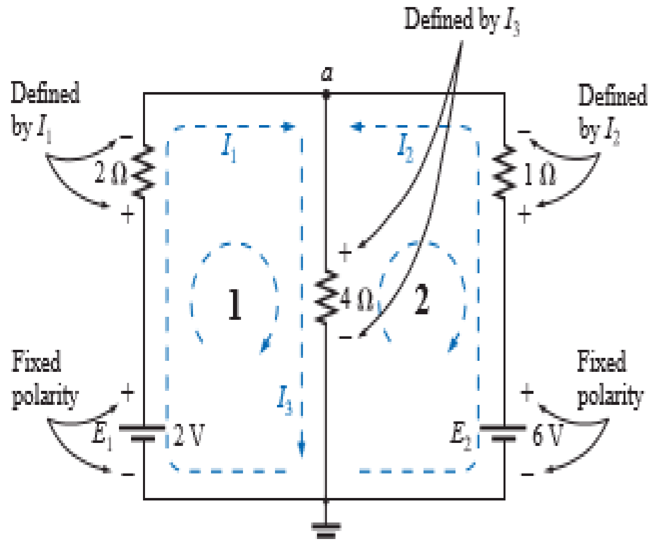
$$\text{loop 1:} \quad \Sigma_C V = \underbrace{+E_1}_{\text{Rise in potential}} \underbrace{- V_{R_1} - V_{R_3}}_{\text{Drop in potential}} = 0$$

$$\text{loop 2:} \quad \Sigma_C V = \underbrace{+V_{R_3} + V_{R_2}}_{\text{Rise in potential}} \underbrace{- E_2}_{\text{Drop in potential}} = 0$$

and

$$\text{loop 1:} \quad \Sigma_C V = \underbrace{+2\text{ V}}_{\substack{\text{Battery}\\\text{potential}}} \underbrace{-(2\ \Omega)I_1}_{\substack{\text{Voltage drop}\\\text{across 2-}\Omega\\\text{resistor}}} \underbrace{-(4\ \Omega)I_3}_{\substack{\text{Voltage drop}\\\text{across 4-}\Omega\\\text{resistor}}} = 0$$

$$\text{loop 2:} \quad \Sigma_C V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$



Step 4: Applying Kirchhoff's current law at node *a* (in a two-node network, the law is applied at only one node),

$$I_1 + I_2 = I_3$$

Step 5: There are three equations and three unknowns (units removed for clarity):

$$\begin{aligned} 2 - 2I_1 - 4I_3 &= 0 \\ 4I_3 + 1I_2 - 6 &= 0 \\ I_1 + I_2 &= I_3 \end{aligned} \qquad \text{Rewritten:} \qquad \begin{aligned} 2I_1 + 0 + 4I_3 &= 2 \\ 0 + I_2 + 4I_3 &= 6 \\ I_1 + I_2 - I_3 &= 0 \end{aligned}$$

Using third-order determinants (Appendix C), we have

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = -1\ \mathbf{A}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = 2\ \mathbf{A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = 1\ \mathbf{A}$$

EXAMPLE 2:

Calculate I1,I2,I3 on the network shown in figure ?

The current directions were chosen to match the "pressure" of each battery. The polarities are then added and Kirchhoff's voltage law is applied around each closed loop in the clockwise direction. The result is as follows

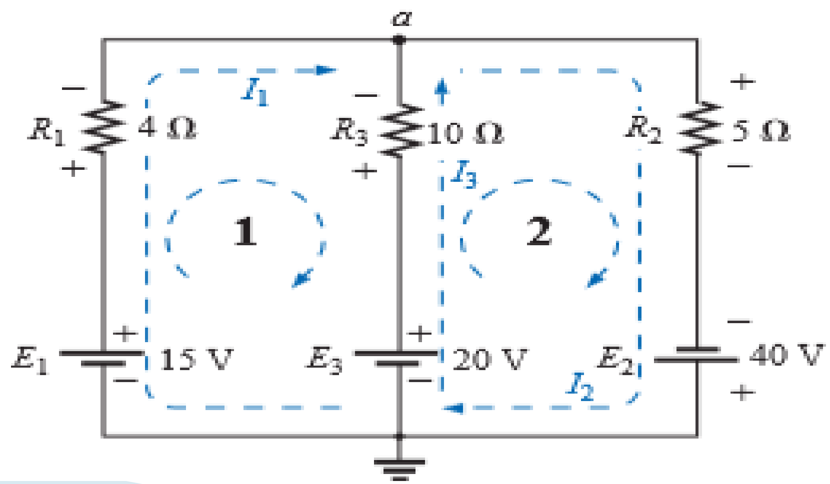
$$\text{loop 1:} \quad +15\text{ V} - (4\ \Omega)I_1 + (10\ \Omega)I_3 - 20\text{ V} = 0$$

$$\text{loop 2:} \quad +20\text{ V} - (10\ \Omega)I_3 - (5\ \Omega)I_2 + 40\text{ V} = 0$$

Applying Kirchhoff's current law at node a,

$$I_1 + I_3 = I_2$$

Substituting the third equation into the other two yields (with units removed for clarity)



$$\left.\begin{aligned} 15 - 4I_1 + 10I_3 - 20 &= 0 \\ 20 - 10I_3 - 5(I_1 + I_3) + 40 &= 0 \end{aligned}\right\}$$ Substituting for I_2 (since it occurs only once in the two equations)

or

$$-4I_1 + 10I_3 = 5$$
$$-5I_1 - 15I_3 = -60$$

Multiplying the lower equation by -1, we have

$$-4I_1 + 10I_3 = 5$$
$$5I_1 + 15I_3 = 60$$

$$I_1 = \frac{\begin{vmatrix} 5 & 10 \\ 60 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 10 \\ 5 & 15 \end{vmatrix}} = \frac{75 - 600}{-60 - 50} = \frac{-525}{-110} = \mathbf{4.773\ A}$$

$$I_3 = \frac{\begin{vmatrix} -4 & 5 \\ 5 & 60 \end{vmatrix}}{-110} = \frac{-240 - 25}{-110} = \frac{-265}{-110} = \mathbf{2.409\ A}$$

$$I_2 = I_1 + I_3 = 4.773 + 2.409 = \mathbf{7.182\ A}$$

revealing that the assumed directions were the actual directions, with I_2 equal to the sum of I_1 and I_3.

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Fundamental of Electrical Engineering

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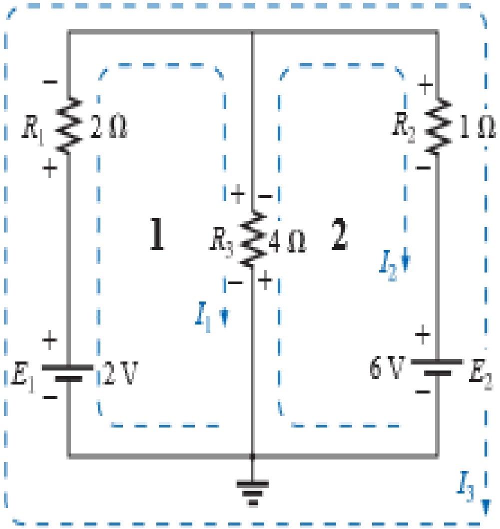
MESH ANALYSIS (GENERAL APPROACH)

EXAMPLE: Compute the current passing through each resister using mesh mrthod?

loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at point a)

$$+2\text{ V} - (2\ \Omega)\,I_1 - \overbrace{(4\ \Omega)\underbrace{(I_1 - I_2)}_{\text{Total current through 4-}\Omega\text{ resistor}}}^{\text{Voltage drop across 4-}\Omega\text{ resistor}} = 0$$

Subtracted since I_2 is opposite in direction to I_1.



loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at point b)

$$-(4\ \Omega)(I_2 - I_1) - (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: The equations are then rewritten as follows (without units for cl

$$\begin{aligned}\text{loop 1:}\quad & +2 - 2I_1 - 4I_1 + 4I_2 = 0\\ \text{loop 2:}\quad & -4I_2 + 4I_1 - 1I_2 - 6 = 0\end{aligned}$$

and

$$\begin{aligned}\text{loop 1:}\quad & +2 - 6I_1 + 4I_2 = 0\\ \text{loop 2:}\quad & -5I_2 + 4I_1 - 6 = 0\end{aligned}$$

or

$$\begin{aligned}\text{loop 1:}\quad & -6I_1 + 4I_2 = -2\\ \text{loop 2:}\quad & +4I_1 - 5I_2 = +6\end{aligned}$$

Applying determinants will result in

$$I_1 = \mathbf{-1\ A} \quad \text{and} \quad I_2 = \mathbf{-2\ A}$$

EXAMPLE :

Find the current through each branch of the network of Figure.

Steps 1 and 2 are as indicated in the circuit. Note that the polarities of the 6- resistor are different for each loop current.

Step 3: Kirchhoff's voltage law is applied around each closed loop in the clockwise direction:

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

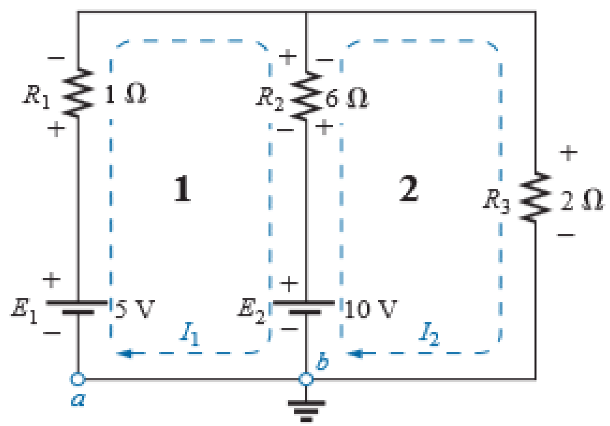
The equations are rewritten as

$$\left.\begin{aligned} 5 - I_1 - 6I_1 + 6I_2 - 10 &= 0 \\ 10 - 6I_2 + 6I_1 - 2I_2 &= 0 \end{aligned}\right\} \begin{aligned} -7I_1 + 6I_2 &= 5 \\ +6I_1 - 8I_2 &= -10 \end{aligned}$$

Step 4:

$$I_1 = \frac{\begin{vmatrix} 5 & 6 \\ -10 & -8 \end{vmatrix}}{\begin{vmatrix} -7 & 6 \\ 6 & -8 \end{vmatrix}} = \frac{-40 + 60}{56 - 36} = \frac{20}{20} = \mathbf{1\ A}$$

$$I_2 = \frac{\begin{vmatrix} -7 & 5 \\ 6 & -10 \end{vmatrix}}{20} = \frac{70 - 30}{20} = \frac{40}{20} = \mathbf{2\ A}$$



Since I1 and I2 are positive and flow in opposite directions through the 6-`Ω resistor and 10-V source, the total current in this branch is equal to the difference of the two currents in the direction of the larger.

$$I_2 > I_1 \quad (2\text{ A} > 1\text{ A})$$

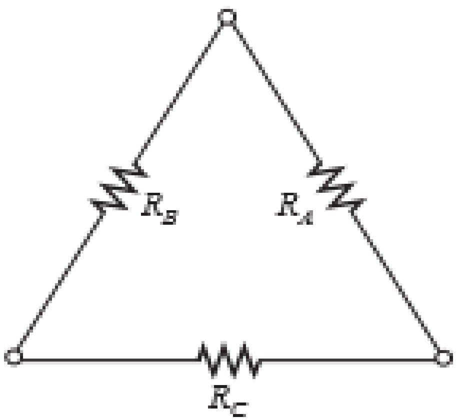
Therefore,

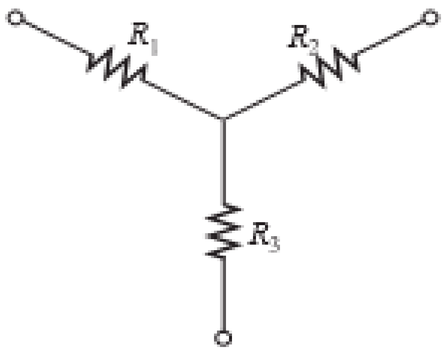
$$I_{R_2} = I_2 - I_1 = 2\text{ A} - 1\text{ A} = \mathbf{1\ A} \quad \text{in the direction of } I_2$$

CONVERSIONS Delta to Star and Star to Delta

➢ **Delta to Star**

Let us first assume that we want to convert the Δ (RA,R B,RC) to theY (R1, R2, R3). This requires that we have a relationship for R1, R2, and R3 in terms of RA,RB, and RC.





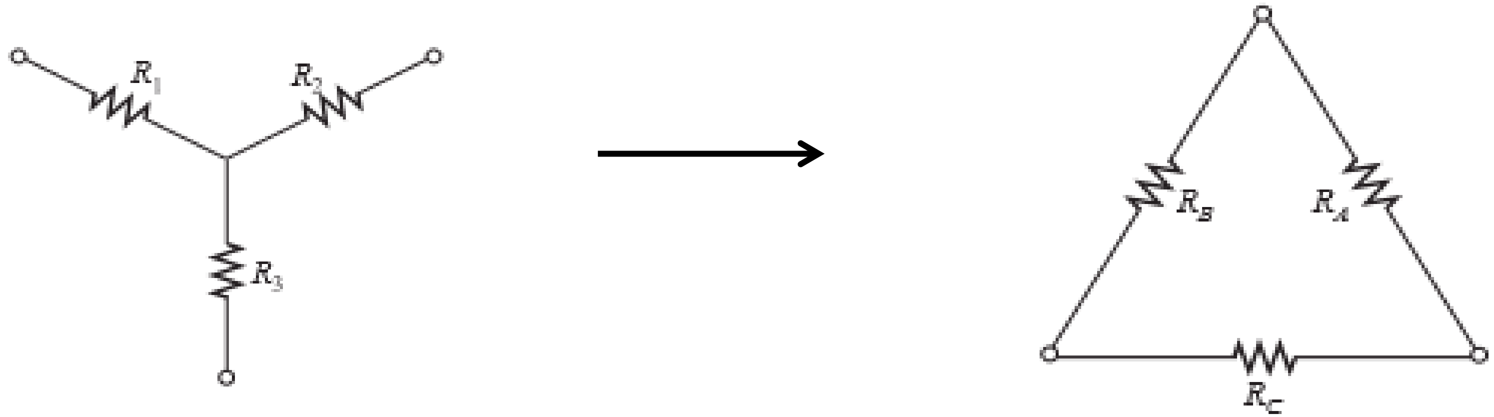
$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

➢ Star to Delta

To obtain the relationships necessary to convert from a Y to a D



$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$